The Calculators on the Insolubles: Bradwardine, Kilvington, Heytesbury, Swyneshed and Dumbleton

Stephen Read

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Abstract

The most exciting and innovative period in the discussion of the insolubles (i.e., logical paradoxes) before the twentieth century occurred in the second quarter of the fourteenth in Oxford, and at its heart were many of the Calculators. It was prompted by Thomas Bradwardine's iconoclastic ideas about the insolubles in the early 1320s. Framed largely within the context of the theory of (logical) obligations, it was continued by Richard Kilvington, Roger Swyneshed, William Heytesbury and John Dumbleton, each responding in different ways to Bradwardine's analysis, particularly his idea that propositions had additional hidden and implicit meanings. Kilvington identified an equivocation in what was said; Swyneshed preferred to modify the account of truth rather than signification; Heytesbury exploited the respondent's role in obligational dialogues to avoid Bradwardine's tendentious closure postulate on signification; and Dumbleton relied on other constraints on signification to give new life to two long-standing accounts of insolubles that Bradwardine had summarily dismissed. The present paper focusses on the central thesis of each thinker's response to the insolubles and their interaction.

Keywords: signification, liar paradox, insolubles, truth; Bradwardine, Dumbleton, Heytesbury, Kilvington, Paul of Venice, Swyneshed.

1 Introduction

This paper considers the work on logic of five of the Oxford Calculators, in particular, their novel approaches to the insolubles, that is, the logical paradoxes. In the thirteenth century, right up until 1320, among the various solutions on the market, two dominated, those of the *cassantes* and the *restringentes*.[1] Nonetheless, although these are often

[1]See e.g., Spade (1987, §§VI-VII).

the only solutions discussed in treatises on insolubles at this time, only two treatises are extant in which *cassatio* is defended, both early;[2] in the others, it is dismissed in favour of *restrictio*, of which several variants were developed. Then in the early 1320s, possibly when Bradwardine was still at Balliol, or in his first year at Merton in 1323-4,[3] he argued powerfully against four versions of *restrictio*, then dismissed two versions of *cassatio* and two further proposals (those of the *mediantes* and the *distinguentes*) before presenting, defending and developing a radically new approach.

The problem should be familiar, often encapsulated in the famous Liar paradox, in the form 'This proposition (sentence, statement, assertion) is not true' or 'What I am saying is false'.[4] The most common form in which it was given by the Calculators was a scenario in which Socrates (a standard term in this context for an awkward customer, without necessarily any historical connotation) says just one thing, namely, 'Socrates says something false'. Did Socrates say something false? Suppose he did: then what he said was false, so it was false that he said something false, so he must have said something true. But if what he said was true, then it was true that what he said was false, so it was false. So what he said was false if and only if it was true, which is paradoxical, and seems to entail that it was both, which is impossible.

The *cassantes* looked to solve the problem by declaring that, despite appearances, Socrates had said nothing. If he'd failed to say anything (coherent) then there was nothing to be true or false and certainly not both. Bradwardine dismissed this with the observation that clearly Socrates had uttered letters, syllables and so on in a coherent way. The suggestion that this was an illusion, as was our reasoning about what he said, is itself paradoxical and not to be taken seriously. The proponent of *cassatio* can reply, however, as we shall see in §6 below, that Socrates may well have made an intelligible sound, but not one coherent enough to constitute a proposition with a truth-value.

The *mediantes* proposed that, though Socrates did say something, it was neither true nor false. We'll consider a version of this view later. The *restringentes* proposed a restriction on the reference (or as they called it, the supposition[5]) of the terms in a proposition, that such terms could not refer to a whole of which they themselves were a proper part. (Note that a term can supposit for itself, in material supposition, just not, they said, for a greater whole of which it is part.) Of the various versions, one banned all such reference to the whole by the part, another only such reference in the presence

[2] See Spade (1975, items IX & XXI).

[3] Fleming (1964, p. 78) dates Bradwardine's treatise to the years 1324-7, but the argument in footnote 10 below suggests it must have been written earlier in the 1320s. The explicit of the Madrid manuscript of Bradwardine's treatise describes its author as 'regens Oxonii': see Bradwardine (2010, p. 2).

[4] Note that throughout this paper, I will use 'proposition' to refer to what the medievals referred to as 'propositiones', that is, concrete token sentences, whether spoken, written or mental.

[5] On the medieval theory of supposition of terms, and how it is both like and unlike the modern notion of reference, see, e.g., Read (2015a, §3).

of privative semantic expressions like 'false' and 'not true'. The former restriction seems implausible in the light of innocuous examples like 'This sentence contains five words'. The latter seems ad hoc unless some independent reason can be given to suppose that such reference is unacceptable, which proved difficult to articulate.

The *distinguentes* made a distinction, saying that Socrates' utterance is true in one sense and false in another. Bradwardine refuted this proposal by the usual method of "revenge":[6] take the sense in which it is said to be true, call it ϕ, and consider the strengthened paradox 'This proposition is not true in sense ϕ'. If it is true in sense ϕ it isn't, and if it isn't true in sense ϕ, then surely it is, for that is what it says. Paradox has returned.

2 Bradwardine's Radical Solution

Bradwardine was born in Hartfield, Sussex, around 1300. He was regent master at Balliol in 1321 and joined Merton College in 1323.[7] He was the first of the Oxford Calculators. He wrote his *De Proportionibus* in 1328 and *De Continuo* in the early 1330s. He became a member of the circle of Richard de Bury, the Bishop of Durham, in 1335. He was Chancellor of St Paul's Cathedral, London, from 1337, and in the 1330s started work on his *De Causa Dei contra Pelagium*, published in 1344. Consecrated Archbishop of Canterbury at Avignon in July 1349, he died at Lambeth from the Black Death within days of his return to England, in August 1349.

Bradwardine's radical idea for a solution to the insolubles can be thought of like this: if Socrates' utterance simply means that what Socrates said was false, we seem to land in contradiction. It turns out that it is true if and only if it is false, so if it is either, it is both. It is implicitly contradictory, so perhaps it really is actually contradictory, signifying not only that it is false, but also, somehow implicitly, that it is true. In that case it must simply be false, since it can't be both. This is his "multiple-meanings" solution: insolubles must mean more than at first appears. Bradwardine, whose later career shows how smart, subtle and deep a thinker he was, came up with a smart, subtle and deep argument to establish this.[8]

Take any proposition, α, he said, which signifies or means that it itself is false—like Socrates' utterance, for example. (Recall that Bradwardine has already dismissed the *restringentes*' suggested ban on self-reference.) Either that is all α signifies, or not. Suppose it is all α signifies, and suppose α is false. As later in his mathematical and

[6] See Bradwardine (2010, ¶5.8). On revenge paradoxes and Bradwardine's use in particular, see Read (2007).

[7] According to Weisheipl (1959, p. 441), Balliol College was founded exclusively for the study of arts, so to proceed to one of the higher faculties, young scholars, such as Bradwardine and later John Wyclif, had to transfer to another college.

[8] See Bradwardine (2010, ch. 6).

theological works, Bradwardine sets out his definitions and postulates explicitly and systematically. First, a proposition is true if and only if it signifies wholly as it is, otherwise false, so every proposition is either true or false. This is his first postulate—bivalence. Everything a proposition signifies must obtain for it to be true; if anything it signifies fails to hold, that is enough to render it false. His second postulate, again consonant with the thought that a proposition might signify many things (and more than might at first appear), is that propositions signify anything that follows from what they signify. If α is false, and assuming it only signifies that α is false, then α signifies wholly as it is, so α is true. But α signifies that α is false, and we've shown that if it's false, it's true, so α must signify that α is true, by the second postulate. So it doesn't signify only that it's false, for we have shown that if all it signifies is that it's false then it also signifies that it's true. It doesn't yet follow that it does signify that α is true, for that only follows from the counterfactual assumption that being false is all that α signifies.

So α signifies more than that α is false—call the extra signification ϕ. So α signifies both that α is false and ϕ. Again, suppose that α is false. Then things are not wholly as it signifies, so something it signifies must fail, i.e., either α is not false or not-ϕ. But α signifies that α is false, so by the second postulate, α signifies that either it's not false or not-ϕ. But it also signifies that ϕ, and it follows from ϕ and 'α is not false or not-ϕ', by logical rules encapsulated in his fourth, fifth and sixth postulates, that α is not false, and so by bivalence, that α is true. So again by the second postulate, it follows that α signifies that α is true. So α signifies both that it's false and that it's true. Thus α is indeed contradictory, and it can't be wholly as α signifies, so α is false. But α was any proposition which signified (*inter alia*) that it itself is false. So we have proved Bradwardine's second thesis, that any proposition which signifies itself to be false, also signifies itself to be true and is false. Similarly, any proposition which signifies that it's not true, signifies that it's false, by the first and second postulates, and so signifies that it is both true and false, and thus is false too. Bradwardine continued in the rest of his treatise to deal with the problem of revenge, and to show how his solution deals with other paradoxes and insolubles.

Two of the manuscripts of his treatise contain what one of them calls "an incidental chapter". It is unclear whether it is by Bradwardine himself, but it is certainly by an adherent of his solution. It opens with the words:

> "After these insolubles were first written ... besides the insolubles presented in the fourth chapter, which the solution of the *restringentes* did not solve, an insoluble was found that is forever incapable of solution in that way. For let A be (one of) 'God exists' (call it [D]) and 'Nothing granted by Socrates is known by you' (call it C), where you do not know if A is [D] or C, and let [B] be 'A is known by you', where only [B] is granted by Socrates."[9]

[9] Bradwardine (2010, ¶A.0, p. 176): *... postquam ista insolubilia fuerunt primo scripta, ... preter*

This is the last of Richard Kilvington's 48 sophisms from his *Sophismata*, written in Oxford shortly after Bradwardine's treatise.[10]

3 Richard Kilvington

Kilvington was the son of a priest from the diocese of York (probably from Kilvington, near Thirsk). He studied at Oriel College in the 1320s. He wrote his *Quaestiones super De Generatione et Corruptione* before 1325, *Quaestiones super Physicam* in 1325-26, *Quaestiones super Libros Ethicorum* between 1326 and 1332, and *Quaestiones super libros Sententiarum Petri Lombardi* in 1334. He became Master of Theology around 1335. He was another member of the intellectual circle round Richard de Bury, the Bishop of Durham, from 1335, and took part in diplomatic missions in the service of Edward III. He was Dean of St Paul's Cathedral by 1354, and died in a second phase of the Black Death in 1361.

Much of Kilvington's *Sophismata* concerns logical puzzles about physical matters, e.g., sophisms about beginning and ceasing. But the last four of the forty-eight sophisms concern epistemic puzzles, culminating in the fiendish Sophism 48. Note the reference to granting in proposition C. This is an allusion to the practice of obligations, and the literature on insolubles and sophisms (and more widely) is ridden through with the language of obligations.

There is still disagreement about what (logical) obligations were for and why they played such a large role in fourteenth-century logic, but at least we now understand how they worked. There is always an Opponent and a Respondent. The Opponent proposes a background scenario (*casus*), which may or may not be counterfactual, but at least establishes the context. Then, in the primary type of obligation, *positio*, a proposition is proposed (the *positum*), usually one that is false in the scenario. The Respondent was expected to admit the obligation provided the *positum* could be true in the scenario, otherwise to reject it. The obligation on the Respondent was then to respond, in accordance with strict rules and without contradicting himself, to a series of propositions put forward by the Opponent. His response could be to grant, to deny or to doubt each of them in turn. Propositions which follow from or are inconsistent with the *positum* and any previously granted propositions or the contradictories of those denied, were said to be "relevant" (*pertinens*), and should be granted if they follow

insolubilia quarto capitulo posita, que solutio restringentium non dissolvit, inventum fuit unum insolubile per istam in perpetuum non dissolvendum. Sit enim a: deus est, que sit b, et nullum concessum a Sorte scitur a te, que sit c, et nescias an a sit b vel c, et sit d ista: a scitur a te, que sola conceditur a Sorte. I've interchanged the labels 'B' and 'D' to fit in with Kilvington's presentation below.

[10] Edited in Kretzmann and Kretzmann (1990b). Kretzmann and Kretzmann (1990a, p. xxvii) date the *Sophismata* between 1321 and 1326; and Jung (2016, §1) before 1325. But the passage cited dates the discovery of Sophism 48 after Bradwardine's *Insolubles.* They must both belong to the early to mid-1320s.

(*pertinens sequens*) and denied if inconsistent (*pertinens repugnans*). The rest were said to be "irrelevant" (*impertinens*), and should be granted if known to be true in the scenario, denied if known to be false in the scenario, and otherwise doubted. This particular scheme of rules was known later as the *responsio antiqua*. A revised scheme was proposed by Roger Swyneshed (whom we will meet later), known as the *responsio nova*,[11] and an idiosyncratic revision to the criterion for relevance was proposed by Kilvington in his discussion of Sophism 47, of which more later.

What is the puzzle with Sophism 48? Recall the constituent propositions:

B 'A is known by you'

A either 'God exists'—D, for short
or 'Nothing granted by Socrates is known by you'—C, for short[12]

where the only proposition granted by Socrates is B. So C is equivalent to 'B is not known by you', that is, 'That A is known by you is not known by you'. C is true, because B is not known by you since you don't know if A is known by you, for you don't know what A is. Then if A is C, A is true. Moreover, if A is D, then A is true and known by you to be true. So either way, A is true, so you do know A (or at least, that A is true), so B is true. But then C is false, for we have shown, and so we know, that A is known by you.[13] But we showed that C was true. Paradox.

Kilvington uses this sophism, among other things, as a springboard to develop his own solution to the insolubles. He writes:

> "I say, then, that no insoluble that is presently under discussion is absolutely true or absolutely false; instead each is true in a certain respect and false in a certain respect." (Kretzmann and Kretzmann, 1990a, p. 142)[14]

He applies this to the Socratic liar, 'Socrates says what is false' (*Sortes dicit falsum*):

> "Suppose, for example, that Socrates says this: 'Socrates says what is false' and nothing else. In that case I say that the term 'what is false' can be taken in one way for what is false in a certain respect and in another way for what is false absolutely. If for what is false in a certain respect, it is

[11]See, e.g., Dutilh Novaes and Uckelman (2016, pp. 389-90).

[12]A is one of C or D, not their disjunction, as Bottin (1973, p. 408), repeated in Bottin (1976, p. 87), seems to think.

[13]This form of inference is sometimes known as Gödel's Rule, from its role, in the form $\vdash A \Rightarrow \vdash \text{Prov}[A]$ (that is, having proved A, we've *ipso facto* proved that A is provable), in the proof of Gödel's theorem. See, e.g., Löb (1955, Rule IV). In epistemic logic, the rule means that if we have proved something we can infer that we know it. In alethic modal logic, it is known as Necessitation.

[14]See Kretzmann and Kretzmann (1990b, p. 146): *Dico, igitur, quod nullum insolubile de quo praesens est locutio est simpliciter verum vel simpliciter falsum; sed quodlibet est verum secundum quid et falsum secundum quid.*

> to be granted. If for what is absolutely false, then it must be denied that Socrates says what is false."[15]

He likens the situation to one Aristotle describes in Book III of his *Nicomachean Ethics* (1110a4-12). To save his ship, the captain may choose to throw his goods into the sea, but he does not do this voluntarily without qualification (or absolutely—*simpliciter*), but only in a certain respect (that is, to save his life and that of his passengers). So the action is voluntary in a certain respect and at the same time not voluntary without that qualification: "I say that one must not grant or deny absolutely that [he] wants to throw his goods into the sea."[16] Similarly, a proposition may be neither true nor false without qualification, but both true in a certain respect and false in a certain respect. In particular, Kilvington says, an insoluble like 'Socrates says what is false'

> "is made true in a certain respect on this account, that the part of the insoluble supposits for the whole of which it is a part."

But in addition

> "it is false in a certain respect because from that insoluble together with a possible hypothesis [*viz* that that is all Socrates says] its opposite follows ... The part, however, does not supposit for its whole in such a way that the insoluble is true absolutely (*simpliciter*) in virtue of the supposition of the part for the whole."[17]

Applied to Sophism 48, Kilvington distinguishes two ways of taking the phrase 'known by you'. D ('God exists') is known by you absolutely, so if A is D, so too is A:[18]

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[15](*loc.cit.*): *Ut, verbi gratia, posito quod Socrates dicat istam: 'Socrates dicit falsum' et nullam aliam. Tunc dico quod iste terminus 'falsum' sumi potest uno modo pro falso secundum quid et alio modo pro falso simpliciter. Si pro falso secundum quid, concedenda est. Si pro falso simpliciter, tunc est negandum quod Socrates dicit falsum.*

[16]Kretzmann and Kretzmann (1990a, p. 142), (1990b, p. 146): *Et dico quod non debet concedi vel negari simpliciter quod ... vult proicere merces in mari.*

[17]Kretzmann and Kretzmann (1990a, p. 142), (1990b, p. 147): *verificatur secundum quid propter hoc, quod pars illius insolubilis supponit pro toto cuius est pars ... et est falsum secundum quid eo quod ex ipso insolubili cum casu possibili sequitur suum oppositum ... Non tamen sic supponit pars pro suo toto ut insolubile sit verum simpliciter propter suppositionem partis pro toto, licet possit esse verum secundum quid propter hoc, quod pars supponit pro suo toto.*

[18]Bradwardine's solution is different, and depends on a further thesis that he argues for in ch.9, that if a proposition signifies that it itself is unknown (to someone) then it also signifies that it is unknown to them that it is unknown to them. (Bradwardine, 2010, ¶9.3). The solution to Sophism 48 is then given in ¶ad A1: "For [B] should be doubted, because if A is [D], [B] is true, and if A is C, [B] is false, because it signifies itself to be unknown to you and consequently it signifies that it is unknown to you that [B] is unknown to you, as is clear in chapter 9. If $\langle C \rangle$ is proposed, it should be conceded; and if it is further argued: C is known to you and [D] is known to you, so A is known to you, the

"But if A is C then A is true in a certain respect and not absolutely. And because I doubt whether A is 'God exists' or C, I doubt both senses of the sophism. And the reason why I say that if A is C, A is true, is this: because if A is C, then it is an insoluble, and each insoluble is true in a certain respect and false in a certain respect and neither true nor false absolutely."[19]

4 William Heytesbury

Writing ten years later, William Heytesbury published a solution very similar to Bradwardine's, using an argument made famous in modern times by Arthur Prior.[20] Heytesbury was a Fellow of Merton College by 1330 and according to a note in one manuscript, wrote his *Rules for Solving Sophisms* (*Regulae*) in 1335.[21] He also wrote the treatises *On Compounded and Divided Senses*, *Sophismata*, *Sophismata Asinina* and *Iuxta Hunc Textum* during the 1330s. He was a foundation Fellow of Queens College at its foundation in 1341, but shortly afterwards returned to Merton. He was Doctor of Theology by 1348, Chancellor of University of Oxford in 1352-4(?) and 1370-2 and died in 1372 or 1373.

Suppose Epimenides really did say 'All Cretans are liars', as Clement of Alexandria claimed,[22] and suppose he was justified in his pessimism about his compatriots—all other Cretan utterances were false. Then if Epimenides' utterance was true it was false (for it's a Cretan utterance); and conversely, if it was false, it must be true (for then all Cretan utterances would be false). That's impossible, so not all other Cretan utterances were false. Simply in virtue of Epimenides saying what he did, something said by a Cretan must be true, but it cannot, it seems, be what Epimenides himself

conclusion should be conceded. And if it is objected: therefore [B] should be conceded, which earlier was doubted, it must be said that it does not follow, but what you concede is a proposition similar to [B]."

[19]Kretzmann and Kretzmann (1990a, p. 139), (1990b, pp. 142-3): *Sed si A sit C, tunc A est verum secundum quid et non simpliciter. Et quia dubito utrum A sit ista 'Deus est' vel C, ideo utrumque sensum sophismatis dubito. Et causa quare dico quod si A sit C, A est verum, est ista: quia si A sit C, tunc est unum insolubile, et quodlibet insolubile est verum secundum quid et falsum secundum quid et neque verum neque falsum simpliciter.*

[20]See Prior (1958).

[21]See Heytesbury (1979, p. 2).

[22]Clement of Alexandria (1991, I 14, p. 66), writing in the second century CE, seems to have been the first to identify Epimenides the Cretan as the author of the famous remark of St Paul's in his *Epistle to Titus* that "Cretans always lie". However, the earliest occurrence of the passage from Epimenides' (lost) panegyric to Minos, written around 600 BCE, is in Callimachus' *Hymn to Zeus*, written in the third century BCE, where Crete is said to possess Zeus' tomb, as well as his birthplace—but gods never die. See Callimachus (2010, pp. 15 and 35 ff.). Epimenides himself was drawing on Hesiod and possibly even Homer.

said. That is in itself paradoxical.

Bradwardine and Heytesbury drew a slightly different conclusion from Prior's. Prior infers that some other Cretan utterance was true. Bradwardine and Heytesbury infer that Epimenides' utterance must mean something more than at first appears. When Epimenides uttered 'All Cretans are liars' he must have said more than just that all Cretans are liars. As we saw, Bradwardine has a subtle argument depending on his second postulate to tell us what this further hidden meaning is. Heytesbury draws back, and exploits the apparatus of obligations to keep silent. Taking the now familiar obligational scenario where Socrates says 'Socrates says something false' and nothing else, he observes:

> "But if [the Opponent] asks what the proposition uttered by Socrates signifies under the scenario other than that Socrates something false, I say to him that the Respondent does not have to settle this or that question, because it follows from this scenario that this proposition signifies other than that Socrates says something false, but that scenario does not specify what ⟨that other signification⟩ is, and so the Respondent does not have to settle that question any further."[23]

Bottin (1985, p. 241) attributes to Bradwardine a distinction between the principal signification of a proposition and its consequential signification. This is tendentious, for the idea appears in Bradwardine (2010, see ¶7.2.5) in the mouth of an opponent, and arguably Bradwardine rejects the distinction. Recall from §2 that while for Bradwardine every proposition that signifies that it itself is false also signifies that it is true, that implicit signification that it is true is not consequential (only) on its signifying that it is false.[24] However, such a distinction became popular in later authors, notably in Heytesbury, who introduces the idea of what the words usually mean (*sicut verba communiter pretendunt*), and distinguishes cases where what they usually mean is all they signify from cases where they signify more.[25]

Given this distinction, Heytesbury presents rules for dealing with insolubles for which he became famous. First, he defines an insoluble scenario:

> "An insoluble scenario is one in which mention is made of some proposition such that, if in that scenario it signifies only what its words usually mean,

[23]Pozzi (1987, p. 240): *Si autem queratur in casu illo quid significabit illa propositio dicta a Sorte aliter quam quod Sortes dicit falsum, huic dicitur quod respondens non habebit illud seu illam quaestionem determinare, quia ex casu isto sequitur quod ista propositio aliter significet quam quod Sortes dicit falsum, sed casus ille non certificat quid illud sit et ideo non habet respondens quaesitum illud ulterius determinare* (my translation—cf. Heytesbury (1979, pp. 49-50)).

[24]See also Read (2016, esp. p. 338).

[25]See Pozzi (1987, p. 238): *. . . si fiat casus de insolubili et cum hoc supponatur quod illud insolubile significet sicut termini illius praetendunt non tamen praecise . . . .*

it follows that it is true and that it is false,"[26]

and secondly, what it is to be an insoluble proposition:

> "An insoluble proposition is one of which mention is made in some insoluble scenario such that, if in that scenario it signifies only what its words usually mean, it follows that it is true and that it is false."[27]

Then, he says in his second rule, if anyone presents an insoluble scenario, and specifies that the insoluble proposition signifies only what its words usually mean, the obligation should in no way be admitted. Only if the Opponent leaves open that the proposition might have an additional signification (that it signifies what the words usually mean, but perhaps not only in that way) should the obligation be admitted, and then the insoluble should be granted, and that it is true should be denied. Thus Heytesbury's position gives rise to the same Moorean paradox as Bradwardine's.[28] For example, in the case of Socrates' sole utterance of 'Socrates says what is false', we first grant that what Socrates says is false and immediately deny that what we granted is true. This situation is in fact not so very unusual in obligations where what is granted is often in fact false and something we certainly don't believe.

Spade, in his 'Study' of Heytesbury's theory in Heytesbury (1979, p. 92), confronts Heytesbury with an insoluble that he claims the theory cannot deal with. First, he defines a neologism, 'firm' (p. 81) to capture the notion Heytesbury expressed by the phrase *ita est sicut communiter pretendunt*, which I am translating as 'being as it usually means'. Take the proposition S, namely, 'It is not as S usually means', for short, 'S is infirm'. Suppose S is firm, that is, it is as S usually means. Since S's usual meaning is that it is infirm, it follows that S is infirm. Hence, by *reductio*, S is not firm. But S signifies that it is infirm. So it is as S usually means, that is, S is firm. Contradiction.

Heytesbury appears to have no response to this, though it is very similar, if not identical, to an insoluble he levels at the theories of his contemporaries whom he criticizes in the first part of his treatise, albeit not by name. Cajetan of Thiene (1494,

[26]Pozzi (1987, p. 236): *Casus de insolubili est in quo fit mentio de aliqua proposione quae, si cum eodem casu significaret praecise sicut verba eius communiter praetendunt, sequitur eam esse veram et eam esse falsam.* Some manuscripts and incunables give that last clause as "from its being true it follows that it is false and vice versa" (*ad eam esse veram sequitur eam esse falsam, et econverso*): see Heytesbury (1979, p. 46) and Heytesbury (1494, f. 6rb).

[27](*loc.cit.*): *propositio insolubilis est de qua fit mentio in casu de insolubili quae, si cum eodem casu significaret praecise sicut verba illius communiter praetendunt, sequitur eam esse veram et eam esse falsam.* Again, some manuscripts give the alternative reading of the consequent mentioned in n.26.

[28]Field (2006, p. 715) notes that Tim Maudlin's solution to the Liar paradox commits him to believing simultaneously that the Liar is not true "while believing that this belief of his is not true." This is like Moore's paradox (see, e.g., Moore, 1993), the apparent absurdity of making a first-person assertion of the form 'p but I don't believe that p'.

f. 7va) identifies these theories as those of the Calculators Roger Swyneshed, John Dumbleton and Richard Kilvington respectively.[29] Swyneshed, for one, not only has a response to Heytesbury's purported counterexample, and Spade's variant, but in fact confronts it directly in the final part of his treatise, having laid the foundation for its solution in the very first paragraph of the work.

5 Roger Swyneshed

Roger Swyneshed is perhaps one of the lesser-known Calculators, not to be confused, as over the centuries he sometimes has been, with his better-known namesake, Richard Swineshead. Roger, who may have studied at Oxford under Bradwardine and Kilvington, wrote treatises on *Insolubles* and *Obligations* between 1330 and 1335 (and also a treatise on *Consequences* now apparently lost). He is also the author of *Descriptiones motuum* (or *De motibus naturalibus*), a treatise on natural changes, including locomotion. He subsequently became Master of Theology (though his lectures on the *Sentences* are also lost) and was another member of Richard de Bury's circle. He was a Benedictine monk at Glastonbury and died about 1365.[30]

Swyneshed is determined not to make any great play with the notion of signification, and although he often uses the phrases 'signify principally' or 'principal signification', by them he seems just to mean what it signifies, or in Heytesbury's phrase, 'what it usually means'. Instead, he varies the truth condition, introducing the idea of a proposition's falsifying itself. Consequently, a proposition might signify as it is and still be false, if it falsifies itself:

> "A proposition falsifying itself directly is a proposition signifying principally as it is or other than it is, relevant to inferring itself to be false. And it is of two kinds. One kind is relevant sufficiently, another relevant insufficiently. Relevant sufficiently is a proposition signifying principally as it is or other than it is from which, signifying in this way, it directly follows or is apt to follow that it is false. An example: let the proposition 'This is false' signify principally that this is false, referring to itself. Then it directly follows 'This is false, therefore, this is false'. And in this way it is relevant sufficiently to

[29] Weisheipl (1968, pp. 202-3) and Spade in Heytesbury (1979, p. 73 n.28), cast doubt on the identification of Dumbleton as the author of the second opinion, since Dumbleton was some years Heytesbury's junior, his treatise on insolubles was arguably composed after Heytesbury's, and in it he discusses and rejects Heytesbury's solution. However, Heytesbury and Dumbleton seem both to have been at Merton in the mid-1330s, and when Dumbleton was named in the foundation statutes of Queens College in 1341, he is cited as '*magister*', so it is reasonable to assume that he had been in Oxford for at least a decade and that he and Heytesbury were mutually aware of each other's ideas as they developed prior to each composing his own treatise. See also Hanke (2016, p. 74 n.10).

[30] See Weisheipl (1964).

> inferring itself to be false."[31]

A proposition can also directly falsify itself insufficiently, if it requires added true premises to do so; and it can falsify itself indirectly if it falsifies another which in turn falsifies the first. With these notions in place, Swyneshed can define truth and falsehood:

> "A true proposition is a proposition not falsifying itself signifying principally as it is either naturally or by an imposition or impositions by which it was last imposed to signify ... A false proposition is an utterance falsifying itself or an utterance not falsifying itself signifying principally other than it is either naturally or by an imposition or impositions by which it was last imposed to signify."[32]

Finally, Swyneshed defines an insoluble:

> "An insoluble as put forward is a proposition signifying principally as it is or other than it is which is relevant to inferring itself to be false or unknown or not believed, and so on."[33]

One might be tempted to infer from these definitions that every proposition, whether or not it is an insoluble, is true or false. But this is to assume that every proposition either signifies as it is or other than it is. For Swyneshed lays down that

> "A proposition is a well-formed indicative utterance significative either naturally or by an imposition or impositions by which it was last imposed to signify complexly."[34]

[31] Spade (1979, pp. 182-3), reprinted in Spade (1988): *Propositio falsificans se immediate est propositio significans principaliter sicut est vel aliter quam est pertinens ad inferendum se ipsam fore falsam. Et illa est duplex. Quaedam est pertinens sufficiens, quaedam est pertinens insufficiens. Pertinens sufficiens est propositio significans principaliter sicut est vel aliter quam est ex qua sic significando immediate sequitur vel est natum sequi ipsam fore falsam. Exemplum: Significet illa propositio 'Hoc est falsum' principaliter quod hoc est falsum, ipsamet demonstrata. Tunc sequitur immediate 'Hoc est falsum; igitur, hoc est falsum'. Et sic illa est pertinens sufficiens ad inferendum se ipsam fore falsam.*

[32] (1979, pp. 185-6): *Propositio vera est propositio non falsificans se principaliter sicut est significans naturaliter aut ex impositione vel impositionibus qua vel quibus ultimo fuit imposita ad significandum ... Propositio falsa est oratio falsificans se vel oratio non falsificans se principaliter aliter quam est significans naturaliter, ex impositione, vel impositionibus qua vel quibus ultimo fuit imposita ad significandum.*

[33] (1979, p. 186): *Insolubile ad propositum est propositio significans principaliter sicut est vel aliter quam est pertinens ad inferendum se ipsam fore falsam vel nescitam vel non creditam, et sic de singulis*, corrected by Pozzi (1987, p. 182).

[34] (1979, p. 185): *Propositio est oratio indicativa congrua naturaliter, ex impositione vel impositionibus qua vel quibus ultimo fuit imposita complexe ad significandum significativa.*

However, it turns out that, in his view, a proposition can signify in some way (*aliqualiter*) but still neither signify as it is nor other than it is. He says as much right at the start of his treatise:

> "A proposition neither signifying principally as it is nor other than it is, that is, which is neither true nor false, is a proposition signifying in some way and that so signifying is relevant to inferring itself not to signify principally as it is, for example, the proposition 'This proposition does not signify as it is', referring to itself, which principally signifies that it itself does not signify as it is. And this similarly, 'Every proposition signifies other than it is', which principally signifies that every proposition signifies other than it is."[35]

This stance allows Swyneshed to deal with the paradox of signification which, we noted, was launched at Heytesbury by Spade. To be sure, it doesn't fit Swyneshed's definition of an insoluble, which must signify as it is or other than it is (as stated in his definition, cited above), but it's still a problematic sophism, as Swyneshed notes in the fifth and final section of his treatise, which aims "to solve some sophisms which appear to be insolubles but are not, e.g., '*A* is known', 'This proposition signifies other than it is', 'That proposition does not signify other than it is', 'This proposition does not signify as it is', and similar ones."[36]

How does Swyneshed deal with this last example? The core idea of Swyneshed's theory was that the criterion of truth be strengthened to exclude propositions which falsify themselves, that is, are "relevant to inferring their own falsity", and the criterion of falsehood weakened to include such propositions. This is recorded in Swyneshed's first iconoclastic thesis:

> "Some false proposition signifies principally as it is."[37]

Socrates' lone utterance of 'Socrates says something false' is an example. It signifies as it is (it's false) because it satisfies Swyneshed's definition of falsehood—it falsifies itself. But the paradox of signification does not, or at least not obviously, falsify itself. Nonetheless, Swyneshed prepared himself for this right at the start of his treatise:

[35](1979, pp. 180-1): *Propositio nec principaliter significans sicut est nec aliter quam est, id est quae nec est vera nec falsa, est propositio significans aliqualiter esse et illa sic significando est pertinens ad inferendum se ipsam non significare principaliter sicut est, sicut haec propositio 'Haec propositio non significat sicut est', demonstrata illa eadem, quae principaliter significet quod ipsa non significat sicut est. Et haec similiter 'Omnis propositio significat aliter quam est' quae principaliter significet quod omnis propositio significat aliter quam est.*

[36](1979, p. 215): *His vero completis superest solvere quaedam sophismata quae apparent insolubilia et non sunt, sicut sint 'a est scitum', 'Ista propositio significat aliter quam est', 'Illa propositio non significat aliter quam est', 'Ista propositio non significat sicut est', et his similes.*

[37](1979, p. 188): *Aliqua propositio falsa significat principaliter sicut est.*

> "A proposition relevant to inferring itself not to signify principally as it is, is one from which, with its being wholly as it is (*cum totaliter sic esse sicut est*), it follows or is apt to follow that it does not signify principally as it is. An example: let the proposition 'This signifies other than it is' signify principally that it signifies other than it is, referring to itself. Then it follows: this proposition signifies other than it is, and it signifies principally like that, hence, it does not signify principally as it is. And thus from it, with its being wholly as it is, it follows that it does not signify as it is. And so it neither signifies other than it is nor as it is."[38]

The same move works for 'This proposition does not signify as it is'. For, as he says, "this follows directly: 'it does not signify as it is, therefore, it does not signify as it is'."[39] So the second leg of Spade's argument against Heytesbury should be denied. To be sure, S (that is, 'S is infirm') is not firm, and it signifies that it is infirm. But it does not follow that it is as it signifies. As Swyneshed says, "it is necessary to add to the premise that it is not relevant to inferring itself not to signify as it is. And if that is added, it should be denied."[40] So S neither signifies as it is nor other than it is, and is neither true nor false.

6 John Dumbleton

As noted, Heytesbury discusses, and rejects, the solutions to insolubles proposed by Swyneshed, by Dumbleton and by Kilvington. He then adopts, reluctantly and with doubts as to there being any thoroughly satisfactory solution, a variant of Bradwardine's. If we turn to Dumbleton's own treatise, we find him discussing and rejecting the approaches of William of Ockham, of Bradwardine, of Swyneshed and of Heytesbury.

John Dumbleton was probably a native of Dumbleton in Gloucestershire. He became Master of Arts at Oxford and Bachelor of Theology at Paris. He was listed as Fellow of Merton College in 1338 and was also a founding Fellow of Queens College in 1341. He studied theology at Paris in the early 1340s and was Fellow of the Sorbonne in the mid-1340s. Returning to Oxford around 1347, he was listed at Merton in 1348.

[38](1979, p. 181): *Propositio pertinens ad inferendum se ipsam non significare principaliter sicut est est talis ex qua cum totaliter sic esse sicut est sequitur vel natum est sequi illam non significare principaliter sicut est. Exemplum: Significet haec propositio 'Haec significat aliter quam est' principaliter quod haec significat aliter quam est, ipsamet demonstrata. Tunc sequitur 'Haec propositio significat aliter quam est; et principaliter sic significat; igitur, haec non significat principaliter sicut est'. Et ita ex illa cum totaliter sic esse sicut est sequitur quod ipsa non significat sicut est. Et ideo nec significat aliter quam est nec sicut est.*

[39](1979, p. 219): *Nam sequitur immediate 'Illa non significat sicut est; ergo, illa non significat sicut est'.*

[40](*loc.cit.*): *Sed oportet addere antecedenti quod illa non foret pertinens ad inferendum se ipsam non significare sicut est. Et si illud addatur, illud est negandum.*

His *Summa Logicae et Philosophiae Naturalis* was left unfinished at his death, probably from the Black Death in 1349.[41]

Dumbleton's treatise on insolubles constitutes a very small part of his long treatise, running to some 400,000 words, entitled *Summa Logicae et Philosophiae Naturalis*. It is preserved in 21 manuscripts, though two omit the first part, on logic. Part I: *Summa Logicae* opens with a short treatise on signification (*De significatione termini*). This is essential for underpinning and defending Dumbleton's approach to the insolubles, for he argues for a version of *cassatio*: insolubles signify in such a way that they fail to express propositions. The reason for this failure, he says, is that an utterance, spoken or written, is a proposition only if it communicates something from the speaker to the hearer.

The treatise opens with the at first surprising statement:

> "No term signifies anything naturally."[42]

The manuscripts all agree in this reading, but in one manuscript the scribe inserts '*in voce vel scripto*' interlinearly after '*terminus*', that is, 'spoken or written'. Dumbleton accepts, along with most medievals, Aristotle's statement in *De Interpretatione* (16a3) that "spoken sounds are symbols of affections in the soul and written marks of spoken sounds." But he restricts the word 'term' to spoken sounds and written marks.[43] They are symbols of concepts (*intentiones*) in the mind (*anima*) or understanding (*intellectio*).[44] Consequently, given that they don't signify naturally, he says, we need

> "to examine how spoken and written terms signify, how they are imposed to signify, how a term is significative of something arbitrarily (*ad placitum*), how we arrive at knowledge of terms, and finally how anything can be

[41] See Sylla (2008).

[42] *De significatione termini*, ch.1: *Nullus terminus aliquam rem significat naturaliter.* All citations from Dumbleton's *Summa Logicae* are preliminary readings from an edition in preparation by Barbara Bartocci and myself.

[43] Weisheipl (1968, p. 204) misunderstands Dumbleton's claim to mean that even concepts signify *ad placitum*, using this to cast doubt on the authenticity of a short treatise on sophisms preserved in five of the nineteen extant manuscripts of Dumbleton's *Summa Logicae* (Weisheipl cites Merton College MS 306, f.1ra, the first folio of the treatise, the third in the manuscript), where it is said: "concepts in the mind do not signify arbitrarily, but naturally" (*intentiones in anima non ad placitum, sed naturaliter significant*). But Dumbleton agrees; he just decides not to call concepts "terms".

[44] Hasse (2014, §3) writes: "In Arabic-Latin translation literature, *intentio* is very often used to render *ma'nâ*, with the consequence that the term *intentio* took on a similarly broad semantic range as its Arabic counterpart. In the writings of Avicenna, *ma'nâ* may mean 'concept', but also 'meaning' of a word, or something 'intelligible' by the intellect, or 'perceptible' by estimation but not by the external senses ... In Averroes' epistemology, the term *ma'nâ* has a specific meaning as the object of memory and a broader meaning as the abstracted content of sensory, imaginative or intelligible forms."

> signified by any term."[45]

Dumbleton's answer is that a term signifies only when, on account of the proper concept (*intentio propria*) which is linked to the term, the hearer or the reader actualizes the true concept of the thing (*intentio vera rei*) which he has stored in his memory. This presupposes that there has already been a first imposition, namely, that a relation has already been established between the proper concept of the term (*propria intentio termini*) and the true or proper concept of the thing (*vera/propria intentio rei*). In other words, a term only signifies when and insofar as our understanding performs an act which consist in recalling to mind or remembering the true concept of the thing by means of the proper concept of the term. In brief, all signification is mediated by concepts, or simple acts of understanding:

> "For a term to signify something incomplexly is to call the concept of the thing to act or to mind through the proper concept of the term itself, and the thing is then said to be signified by this term according to its use ... For a term or terms to signify complexly is to recall the concept of a thing or concepts of other things to a simple act of understanding."[46]

Consequently, Dumbleton draws the conclusion that

> "for a term to signify to someone is nothing other than for that term by its own (or proper) concept to recall the true concept of another thing to mind."[47]

Indeed, what is complex or incomplex (*complexum et incomplexum*) exists only in the mind, for truth and falsity are there alone. Dumbleton cites Averroes' commentary on Aristotle's *Metaphysics* (1027b25-a1), where Aristotle notes that truth and falsity are accidental beings in the mind, and consequently, Averroes comments, composition and division are similarly only in the mind.[48] The term 'is' (*est*) is not itself the copula, but it prompts the mind to conjoin or compose (*copulare*):

[45] *De significatione termini*, ch.2: *Cum igitur termini in voce nec in scripto ex natura eorum rem complexe nec incomplexe significant, ut probatum est, restat exprimere primo, qualiter termini in voce et in scripto significant. Secundo, qualiter imponuntur ad significandum. Tertio, qualiter terminus cuiuscumque rei ad placitum sit significativus. Quarto, quomodo in cognitionem terminorum deveniemus. Et ultimo, qualiter quelibet res a quolibet termino potest significari.*

[46] (*loc.cit.*): *Primi membri descriptio talis est: terminum rem aliquam significare incomplexe est ipsius termini propria intentione intentionem rei ad actum sive ad memoriam reducere que res per talem terminum iuxta usum dicitur significari ... Sequitur descriptio secundi membri que talis est: terminum sive terminos significare complexe est intentionem rei vel intentiones rerum aliarum ad actum simplicem intelligendi reducere.*

[47] (*loc.cit.*): *Ex istis patet una conclusio: quod apud aliquem terminum significare non est aliud nisi istum terminum per intentionem propriam intentionem alterius rei veram ad memoriam reducere.*

[48] *De significatione termini*, ch.4: *Ideo concluditur quod complexum ⟨et incomplexum⟩ solum habent esse in anima; ista sentencia predicta habetur ab Aristotele sexto Metaphysice textu commenti ultimi,*

> “Although the term ‘is’ prompts us to put things together, nonetheless it is not the copula, but just as things perceived prompt us to put things together, such is this term ‘is’ with respect to us, and yet it is not the copula, but prompts the mind to compose by its own concept for what it perceives.”[49]

This analysis of the notion of signification leads Dumbleton to the first conclusion of his treatise on insolubles:

> “The first of the conclusions is this: a term signifies only while and insofar as there is actual comprehension by it.[50] This is made clear like this: no term signifies from its own nature, nor as a result of an imposition, but the terms alone make us comprehend other things; therefore, if a term signifies, it only signifies while and insofar as ⟨there is actual comprehension by it⟩. Again, a concept only signifies while there is actual comprehension through it, but a concept signifies to us naturally prior to and more than a term, and the signification of a term is from the signification of the concept. Therefore *a fortiori* a term only signifies in so far as actual comprehension is had by it.”[51]

The upshot is that Dumbleton’s solution to the insolubles is part of a broader analysis of linguistic understanding and communication. Not only do insolubles fail to constitute propositions, so too do many other expressions.

This is in marked contrast to Bradwardine’s and Heytesbury’s solutions, which apply specifically and only to insolubles. Kilvington’s and Swyneshed’s solutions are also a consequence of their reflections on wider issues: e.g., Swyneshed notes (Spade, 1979,

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ubi ponit quod verum et falsum sunt entia per accidens et sunt in mente et non in rebus, super quo dicit Commentator quod illud quod est in cogitatione est compositio et divisio absque eo quod extra animam et illa encia non essent nisi esset anima. See Averroes (1562, ff. 151v-152v).

[49](*loc.cit.*): *Sed licet talis terminus ‘est’ movet nos ad componendum non tamen est copula, sed sicut sensibilia movent nos ut componamus, sic iste terminus ‘est’ respectu nostri et tamen non est copula, sed movet ut anima copulet pro illo quod percipit propria intentione.*

[50]We could perhaps read between the lines something like ‘only while and insofar as, through the proper concept of the term (or, the term’s own concept—*propria intentio termini*), the term recalls to the intellect the true concept of the thing (*intentio vera rei*) . . . ’.

[51]*Insolubilia*, ch.8 (or 13): *Conclusionum prima est hec: nullus terminus significat nisi dum et quatenus est actualis comprehensio per illum. Hec sic patet: nullus terminus significat ex natura propria nec inducta ab imponente, sed solum termini faciunt nos alias res intelligere; ergo si terminus significat, non significat nisi dum et quatenus etc. Item: intentio non significat nisi dum est actualis comprehensio per eandem, sed prius naturaliter et magis significat nobis intentio quam terminus, et termini significatio est ex significatione intentionis; ergo a multo maiori terminus non significat nisi quatenus habetur actualis comprehensio per illum.* (Some manuscripts continue the chapter numbering from the end of *De significatione termini*, others start numbering afresh, yet others omit chapter numbers entirely.)

p. 191) that his argument that sophisms such as 'This proposition signifies other than it is' are neither true nor false likens them to future contingents such as Aristotle's example of the possible future sea-battle, similarly neither true nor false. Moreover, Kilvington's diagnosis of insolubles is an off-shoot of his revision of the rules for obligations, realising that not only must one grant propositions one knows to be false and deny some that one knows to be true, one must also express doubt about some that one actually knows to be true or to be false, evaluating irrelevant propositions as if the *positum* was true.[52]

Dumbleton defines insolubles as propositions which would be both true and false if one were to admit some apparently possible scenario.[53] He diagnoses the problem as arising from the presence of terms like 'truth', 'falsehood', 'proposition' and so on, which signify complex thoughts. Obligational scenarios should only be admitted if there is no circle of priority involved. Take 'A truth exists' (*Verum est*), given the scenario that this is the only proposition. The term 'truth' must stand in some way for a thought, according to his third and fourth conclusions. Although 'truth' is not itself a proposition, what it stands for must be a thought, or mental complex:

> "The third conclusion is this: no proposition is comprehended by a simple concept in the mind, but by a complex thought ... The fourth conclusion, following from the third, is this: the subject or predicate of any proposition signifying a complex thought is itself a proposition, and a proposition corresponds to one or other or both."[54]

But the proposition to which 'truth' corresponds must be a second proposition, different from the proposition 'A truth exists' itself. Thus the scenario posited, where 'A truth exists' is the only proposition, must be rejected as impossible. Recall from §3 that the Respondent in an obligation should admit a scenario and *positum* only if it is possible. This is not to deny that 'A truth exists' signifies in some way. But its signifying in that way is not sufficient for it to count as a proposition. Thus the proposed scenario is inherently contradictory and so, being impossible, should not be admitted.

The anonymous author of an epitome (*compendium*) of Dumbleton's *Insolubles* "approved according to usage at Oxford" (*accepta secundum usum Oxonie*), edited in Bottin (1980, pp. 25-32), divides insolubles, or rather insoluble scenarios, into six groups (*ordines*), each with a rule for how to respond to them. In four of these groups, the scenario should simply not be admitted. These are the first group, where it is

[52]See Read (2015b, p. 401-4).

[53]*Insolubilia*, ch.13 (or 18): *Descriptio insolubilis talis est: insolubile est propositio que ex apparenti casu possibili admisso concluditur esse vera et falsa.*

[54]*Insolubilia*, chs.10-11 (or 15-16): *Tertia conclusio est hec: nulla propositio in anima per intentionem simplicem, sed per complexum comprehenditur ... Quarta conclusio sequens ex tertia est hec: cuiuscumque propositionis significantis pro complexo subiectum vel predicatum est propositio et alteri vel utrique propositio correspondet.*

assumed in the scenario proposed that there is only one proposition, whose extreme (subject or predicate) supposits for a complex thought (*complexum*), e.g., 'A truth exists', 'A falsehood exists'; the second, where a proposition is referred to (*demonstrari*) by its own subject or predicate, e.g., 'It's not as this proposition signifies'; the third, where there is a circle of self-reference, e.g., where A refers to B and C, saying one of them is true, while C refers to A and B, saying that not both of them are true; and the sixth group, where it is proposed that someone believes just one proposition whose extreme supposits for a complex thought, e.g., where Socrates believes 'Socrates is deceived'. Although not spelled out explicitly, the idea is clearly that, although there is an utterance involved which signifies in some way, the order of priority results in a regress whereby its signification is not well founded and the utterance fails to constitute a proposition. So the scenario must be rejected in proposing that the utterance does constitute a proposition.

The fourth and fifth groups are different. In these cases, the scenario may be admitted, but the paradoxical reasoning should be rejected. The fourth group consists of contradictory sets of propositions like the third, but where the circle of self-reference is not vicious or regressive, e.g., where C is 'Every truth is one of these', referring by 'one of these' to two true propositions A and B, by hypothesis, the only other propositions that exist. If C is true, then it's false, for C is not one of A and B; while if C is false, it must be true, for by hypothesis, it is the only proposition besides A and B. The solution proposed appears to be a kind of *restrictio*: even if C is true, it cannot be included under the signification of its subject 'truth'—else there would be a regress and C would not be a proposition at all. C is true, for all it really says is that each of A and B is one of A and B.

The fifth group concerns cases where someone says, or hears or reads, a proposition whose extreme supposits for a complex thought, e.g., if Socrates says 'Socrates says a falsehood'. Once again, the scenario may be admitted, but the reasoning is faulted by appeal to *restrictio*:

> "When it is argued: 'Socrates says a falsehood and A ['Socrates says a falsehood'] signifies only that Socrates says a falsehood, therefore A is true', I deny the inference and one may respond in this way to every insoluble of the fifth group by admitting everything up to where we reach the argument just denied or one like it, which should be denied. The reason why this inference is not valid is this, that in proposition A the predicate is a term suppositing for a statement and no such term can supposit for that of which it is the subject or predicate, hence it is required that it supposit for some other proposition. If the predicate 'falsehood' in that proposition 'Socrates says a falsehood' supposits for a proposition other than 'Socrates says a falsehood', the proposition 'Socrates says a falsehood' is false, because it

signifies that Socrates says a proposition which he does not say."[55]

Thus what Socrates says is false, but the further inference that it is true is vlocked. If a part could supposit for a whole of which it is part, we would have a vicious circularity.[56] That would lead to a breakdown in communication. But communication, we have seen, is for Dumbleton essential to utterances constituting propositions. Thus Dumbleton's solution is a subtle combination of *cassatio* and *restrictio*, by which paradox is averted.

Whether the summary "approved according to usage at Oxford" is an accurate account of Dumbleton's position, and it really is such a combination of *cassatio* and *restrictio*, remains to be seen when Dumbleton's own treatise has been edited and analysed.

7 Conclusion

Thomas Bradwardine's work initiated not just a revolution in mathematical physics, but a riot of responses to the logical paradoxes, the so-called "insolubles".[57] Most of the big names amongst the Calculators cut their teeth on their subtlety and difficulty. As Bradwardine (and others) observed, they are called "insoluble" not because they are impossible to solve, but because of their sheer difficulty.[58] Their attempts at

[55] Bottin (1980, p. 30), corrected against ms Padua 397 ff. 119r-v: *Et tunc ad argumentum quando arguitur: 'Sortes dicit falsum et a sic precise significat quod Sortes dicit falsum, igitur a est verum', nego consequentiam et ita respondetur ad omne insolubile quinti ordinis admittendo totum usque quo deveniat ad illud argumentum iam negatum vel consimile, ⟨quod⟩ debet negari. Causa quare talis consequentia non valet est ista, quia in a propositione predicatum est terminus supponens pro complexo et nullus talis terminus potest supponere pro illa cuius est subiectum vel predicatum, ideo oportet quod supponat pro alia quacumque propositione. ⟨Si pro⟩ alia ab illa 'Sortes dicit falsum' supponit illud predicatum 'falsum' in illa propositione, 'Sortes dicit falsum', falsa est illa propositio, 'Sortes dcit falsum', quia significat Sortem dicere propositionem quam non dicit.*

[56] One can't but be reminded here of Bertrand Russell's "Vicious Circle Principle", which he took from Poincaré as a solution to the paradoxes in Russell (1973, p. 198, published in 1906): "whatever in any way concerns *all* of *any* or *some* (undetermined) of the members of a class must not be itself one of the members of a class." It received its definitive statement in Russell and Whitehead (1927, p. 37): "An analysis of the paradoxes to be avoided shows that they all result from a kind of vicious circle. The vicious circles in question arise from supposing that a collection of objects may contain members which can only be defined by means of the collection as a whole."

[57] Around 1360, Ralph Strode wrote: "The opinions mentioned were in former times those of the old [logicians], who understood little or nothing about insolubles. After them there arose the prince of modern philosophers of nature, namely Master Thomas Bradwardine. He was the first to discover something worthwhile about insolubles" (*Predicte namque opiniones fuerunt antiquitus antiquorum, qui parum vel nihil de insolubilibus recte sapuerunt. Post quos surexit princeps modernorum physicorum videlicet magister thomas braduardyn qui aliquid quod valuerit de insolubilibus primitus adinvenit*). Ms Erfurt *Biblioteca Amploniana* Quarto 277, f. 3vb, cited in Spade and Read (2017, §3.1).

[58] Bradwardine (2010, ¶2.1): "'insoluble' is here taken [not in the first way, "for what is in no way soluble" but] in this way: an insoluble is a difficult paralogism *secundum quid et simpliciter* resulting

solution led them to explore fresh ideas about truth, signification (or meaning) and logic, the latter in a broad sense covering reasoning and argumentation, as developed in obligational disputations.

Bradwardine's "multiple-meanings" solution built on the idea that each insoluble proposition is somehow implicitly contradictory, and by appeal to his closure postulate, that every proposition signifies anything that follows from what it signifies, he spelled out clearly how insolubles contradict themselves and must all be false. This involves a natural rethinking of the definition of truth. For the medievals, following Augustine, truth is an achievement, a zenith, such that any falling short results in falsehood.[59] Bradwardine infers that everything signified by a proposition must obtain for it to warrant the epithet 'true'. Insolubles signify both that they are false and that they are true, and nothing can satisfy that demand. That is what blocks the second leg of the paradox argument in §1, from supposing the liar proposition is false to inferring that it must therefore be true. Not so: that it is false is not enough to show that it is also true. So the insolubles are simply false.

Although broadly influential, Bradwardine's solution itself commanded fairly narrow support. Robert Eland, for example suggests the best solutions available were Bradwardine's and Heytesbury's, and one must choose between them.[60] Heytesbury's is a variant of Bradwardine's, dropping the closure postulate and thereby the proof that every insoluble signifies its own truth as well as its own falsity, relying instead on the simple but surprising conclusion that every insoluble must have some secondary, unspecified, signification, on pain of contradiction. Nonetheless, the upshot is the same: insolubles are false, since in some way they fall short of the demands of truth.

Like Heytesbury's, Kilvington's approach was also introduced in the context of

from the reflection of some act on itself with a privative determination" (*Insolubile autem sic acceptum describitur hoc modo: insolubile est difficilis paralogismus secundum quid et simpliciter ex reflexione alicuius actus supra se cum determinatione privativa proveniens*).

[59] Aquinas (*Summa Theologica* 1a q.16 art.1 ad 1am) quotes Augustine: "Truth is a supreme likeness to a principle without any unlikeness" (*veritas est summa similitudo principii, que sine ulla dissimilitudine est*), and Buridan (1994, p. 91) writes: "If there were a proposition which however it signified, so it was and signified in no way other than it was, it would be true. And as soon as it signified in some way other than it was, it would cease to be true and would begin to be false, so it is that, just as for something to be said to be the greatest heat, it is necessary that it does not have any degree of coldness, similarly for a proposition to be true, it is necessary that it signifies in no way other than it is" (*si esset aliqua propositio que qualitercumque significaret ita esset et nullo modo significaret aliter quam esset, ista esset vera. Et quam cito significaret aliqualiter aliter quam esset, desineret esse vera et inciperet esse falsa, ita quod sicut ad hoc quod aliquid dicatur summe calidum, requiritur quod non habeat aliquem gradum frigiditatis, ita ad ⟨hoc⟩ quod aliqua propositio sit vera, requiritur quod nullo modo significet aliter quam est*).

[60] See Read and Thakkar (2016, p. 173): "So ⟨these⟩ two responses [sc. Bradwardine's and Heytesbury's] are better than the others for solving insolubles. So the respondent should choose one of them for his solution to the insolubles" (*Ideo duae responsiones sunt meliores aliis ad insolubilia solvenda. Eligat ergo respondens unam istarum pro sua solutione ad insolubilia*).

obligational disputations. Given a certain scenario, e.g., where Socrates' only utterance is 'Socrates says something false', should we grant or deny what he says? It's not that simple, he replies. Strictly speaking, it's neither true nor false without qualification. But it is true in a certain respect, because what he said was false, so we should grant it, and deny that it is (simply) false; and it's also false in some respect, for we have granted what he said, so we should deny that it is (simply) true.

Although Swyneshed had his own radical approach to obligations, he takes a much more direct approach to the insolubles, one which makes no play with any disputational context. Rather, he says, we should recognise that some propositions falsify themselves, and accept that if so, they really are false. This means revising the account of truth and falsity, so that a proposition is said to be true only if it does not falsify itself (and signifies as it is). But the paradoxes of signification require us to go further, and accept that some propositions are neither true nor false, and indeed neither signify as it is nor other than it is.

Finally, Dumbleton takes up Bradwardine's challenge where Heytesbury left off, and looks more closely at the very notion of signification. Propositions don't have meaning in themselves or naturally, but only by being used in communication. It's by that practice of communication and comprehension that they come to be signs of mental complexes. But ultimately, they must lead back to simple concepts, on pain of regress. Problematic in this regard are terms which are themselves signs of complex thoughts. If they were signs of complex thoughts of which they themselves are part, there would be a regress and their signification would be nugatory, failing to constitute a proposition. This is so in particular for the insolubles, which are either false, if, to avoid circularity, such terms are restricted to supposit only for other propositions; or, without such a restriction, fail to express meaningful propositions.

The disparateness of the varied solutions proposed by the Calculators might suggest that no progress was made. But this is to misconstrue the nature of progress in philosophy, which often comes not from narrowing the options available but by broadening them, and through the conceptual revisions and deeper understanding that come from this. The Calculators made the second quarter of the fourteenth century a lively and disputatious time for the discussion of insolubles.

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