Thomas Bradwardine and Epistemic Paradox*

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Abstract

The most famous epistemic paradox is Fitch's paradox. In it, Frederic Fitch offered a counterexample to the Principle of Knowability (PK), namely, that any true proposition can be known. His example is the proposition that some proposition is true but not known. This proposition is not paradoxical or contradictory in itself, but contradicts (PK), which many have found appealing. What is really paradoxical is any proposition which says of itself that it is true but unknown. Thomas Bradwardine, writing in the early 1320s, developed a solution to the semantic paradoxes (*insolubilia*) based on a closure principle for signification: every proposition signifies whatever is implied by what it signifies. In ch. 9 of his treatise, he extends his account to deal with various epistemic paradoxes. Comparison of Fitch's paradox with one of these paradoxes, the Knower paradox ('You do not know this proposition') explains the puzzlement caused by Fitch's paradox. Bradwardine argues that the Knower paradox signifies not only its own truth, but signifies also that it is not known that it is not known, and so is false, since it is known that it is not known. However, his argument is flawed and a different argument for its falsehood is required.

1 Fitch's Paradox

Probably the most famous epistemic paradox is Fitch's paradox. Frederic Fitch produced an example which contradicts the Principle of Knowability (PK, for short), namely, that any true proposition can be known.[1] Fitch's counter-example is the proposition that some proposition is true but not

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[1]Fitch [1963, p. 138].

known. Actually, Fitch's claim is a corollary, attributed to an anonymous referee,[2] of Theorem 1: no proposition of the form $p \wedge \neg\alpha p$ can belong to α, any class of true propositions closed under analogues of (K')—from left to right—and (T) below. The corollary reads: "For each agent who is not omniscient, there is a true proposition which that agent cannot know." This proposition is not paradoxical or contradictory in itself, but contradicts (PK), which many have found appealing. Clearly there are truths which we don't actually know. Fitch showed that this fact is incompatible with (PK)—if not all truths are known, then not all truths can be known.

Epistemic logic is based on two principles, first, the closure of knowledge under known consequence:

If one knows that if p then q then if one knows that p, one knows that q.

In symbols (with $\mathbf{Kn}(p)$ for 'one knows that p'):

$$(K) \qquad (\forall p)(\forall q)[\mathbf{Kn}(p \to q) \to (\mathbf{Kn}(p) \to \mathbf{Kn}(q))].$$

This is weaker than, and entailed by, the implausible principle that knowledge is closed under consequence *tout court*. (K) entails the principle of the distribution of knowledge over conjunction:

One knows that p and q if and only if one knows that p and one knows that q.

In symbols:

$$(K') \qquad (\forall p)(\forall q)[\mathbf{Kn}(p \wedge q) \leftrightarrow (\mathbf{Kn}(p) \wedge \mathbf{Kn}(q))].$$

At least, (K') follows from (K) provided one knows such trivial logical laws as that if p and q then p and that if p then if q then p and q. The proof is immediate, by Adjunction (from $p \to q$ and $p \to r$ to $p \to q \wedge r$) and Importation (from $p \to (q \to r)$ to $(p \wedge q \to r)$.

The second principle is that knowledge is factive, that only truths can be known; in symbols:

$$(T) \qquad (\forall p)[\mathbf{Kn}(p) \to p].$$

Now suppose that every truth is knowable (where '$\Diamond$' stands for possibility):

$$(PK) \qquad (\forall p)[p \to \Diamond\mathbf{Kn}(p)].$$

Applying the Principle of Knowability, (PK), to Fitch's example (some proposition p such that it is true that p and $\neg\mathbf{Kn}(p)$), we get:

$$p \wedge \neg\mathbf{Kn}(p) \to \Diamond\mathbf{Kn}(p \wedge \neg\mathbf{Kn}(p)) \qquad (*)$$

[2] The referee is now known to be Alonzo Church: see Murzi and Salerno [2009].

$$
\begin{array}{llll}
\text{But} \quad \mathbf{Kn}(p \wedge \neg\mathbf{Kn}(p)) & \rightarrow & \mathbf{Kn}(p) \wedge \mathbf{Kn}\neg\mathbf{Kn}(p) & \text{by } (K') \\
& \rightarrow & \mathbf{Kn}(p) \wedge \neg\mathbf{Kn}(p) & \text{by } (T)
\end{array}
$$

a contradiction. So

$$\neg\mathbf{Kn}(p \wedge \neg\mathbf{Kn}(p))$$

whence by Necessitation

$$\neg\Diamond\mathbf{Kn}(p \wedge \neg\mathbf{Kn}(p)) \tag{**}$$

From (*) and (**) we can infer

$$\neg(p \wedge \neg\mathbf{Kn}(p)),$$

i.e., there can be no such proposition p which is both true and unknown—given (PK), that no truth is unknowable. Since there clearly are truths we don't know, (PK) must be given up—not all truths can be known.

Of course, if one knew such a truth to be true, it would not be an unknown truth. So it might seem impossible to give an actual example. Consider, however, the proposition, 'This proposition is true but unknown'. If it were known, it would be true, and so both true and unknown. So by *reductio ad absurdum*, it cannot be known, hence, it seems, both true and unknown. But that is a fallacy, as we will see later. In fact, that proposition is false, not because it is known, but because it's known to be unknown.

Now consider the simpler proposition, 'This proposition is unknown'. This the the Knower Paradox. By the same reasoning, it is unknown, and hence true, so it seems to be an example of an unknown truth. However, we have just proved that it is true, so we do know that it is true. Hence it's both known and unknown, and both true and false. Paradox.

2 Bradwardine on Paradox

Writing in the early fourteenth century, Thomas Bradwardine discussed such epistemic paradoxes, along with semantic paradoxes such as the Liar, 'This proposition is false'. Bradwardine was the first of the Oxford Calculators, who made dramatic advances in mathematical physics and kinematics. Before that, as a young man, Bradwardine wrote a treatise *On Insolubles*, paradoxes such as the Liar and the Knower. Later he wrote a famous defence of the Augustinian doctrine of salvation only through grace. He died of the Black Death in London in 1349, shortly after being created Archbishop of Canterbury.

Bradwardine's solution to the semantic and epistemic paradoxes is based on another closure postulate, the closure of signification under consequence:

A proposition signifies whatever follows from what it signifies.

In symbols (reading '$\mathbf{Sig}(s, p)$' as 's signifies that p':

$$(P2) \qquad (\forall s)(\forall p)(\forall q)[(p \to q) \to (\mathbf{Sig}(s, p) \to \mathbf{Sig}(s, q))].$$

This is Bradwardine's famous second postulate (hence the designation '$P2$'). What Bradwardine in fact wrote is:

> "The second ⟨postulate⟩ is this: Every proposition signifies or means as a matter of fact or absolutely ⟨respectively⟩ everything which follows from it as a matter of fact or absolutely."[3]

Paul Spade [1981, p. 120] takes Bradwardine literally here, and attributes to him what he termed "Bradwardine's Principle", (BP):

(BP) If p only if q, then P signifies that q,

where the name replacing 'P' names the sentence replacing 'p'. But this literal interpretation has a number of unfortunate consequences. First, Spade has to revise Bradwardine's argumentation, since (BP) is inadequate to establish Bradwardine's main theses. Secondly, Spade has to attribute to Bradwardine a further principle, the "Converse Bradwardine Principle", (CBP):

(CBP) If P signifies that q, then p only if q.

However, as Spade observes, (BP) and (CBP) together allow paradox to re-emerge, and so lead to a failure of the solution. Such an outcome should give one pause, in particular to reflect on (BP).

(BP) is not a closure principle. Spade derives the closure principle $(P2)$ from (BP) and (CBP), "even though," Spade notes [1981 p. 124], Bradwardine "very likely never went through the fourteen-step argument" which Spade gives. Certainly not: that argument uses (BP), twice, and (CBP), principles Bradwardine never employs. Indeed, as Spade admits, he never even states (CBP), and it is used by Spade only to recover steps which do not follow by (BP) alone but for which Bradwardine actually uses $(P2)$. Hence, charity suggests we should interpret his statement in ¶6.3 as elliptical for $(P2)$, and not as endorsing (BP).

Indeed, the whole interest of Bradwardine's solution depends on taking his central claim to be $(P2)$, not (BP). For the crucial step in the solution is

[3] Bradwardine [2010, ¶6.3]: "*Secunda ⟨suppositio⟩ est ista: quelibet propositio significat sive denotat ut nunc vel simpliciter omne quod sequitur ad istam ut nunc vel simpliciter.*"

to block one leg of the reasoning in the establishment of paradox. Consider the central example of a semantic paradox which Bradwardine discusses at length in ch. 7. Suppose Socrates utters only:

> Socrates utters a falsehood,[4]

call it A. The familiar reasoning goes as follows: suppose A is true. Then by (**Tr**-out)—see below—Socrates utters a falsehood, so, since A is Socrates' only utterance, A is false and hence, by Contravalence, not true. So by *reductio ad absurdum*, A is not true, whence by Bivalence, A is false. But Socrates uttered A, so by (**Tr**-in), A is true. Hence A is both true and false—contradiction, and paradox.

There are four principles used here: (**Tr**-in), (**Tr**-out), Bivalence and Contravalence. Bradwardine accepts the third of these in his first postulate:

($P1$) Every proposition is true or false.[5]

Moreover, he clearly takes it that no proposition is both true and false, which I will refer to as Contravalence. But Bradwardine does not endorse (**Tr**-in) or (**Tr**-out), at least, not in the simple forms in which they were used in the paradoxical reasoning. (**Tr**-in) and (**Tr**-out) can be expressed in Spade's notation as follows:

(**Tr**-in) If P signifies that q, then q only if $\mathbf{Tr}(P)$

(**Tr**-out) If P signifies that q, then $\mathbf{Tr}(P)$ only if q.[6]

Given that A signifies that Socrates utters a falsehood, it follows from (**Tr**-in) and (**Tr**-out) that A is true if and only if Socrates utters a falsehood.

What (**Tr**-in) fails to recognise is that A may signify more than just that Socrates utters a falsehood. Implicit in ($P2$) is the possibility, indeed, likelihood, nay, certainty, that signification will be pluralistic and manifold, that any proposition will signify many things, to be precise, everything that follows from what it signifies. (**Tr**-out) is fine: whatever a true proposition signifies must be the case. But (**Tr**-in) is too strong: it is not sufficient for

[4] Bradwardine [2010, ¶7.1]: "*Incipiamus ergo ab affirmativis simplicibus et a communissimo inter illa negotium inchoemus. Et circa illa multa dubitabilia movebuntur que cuilibet insolubili poterunt applicari. Dicat ergo Sortes tantum istam: Sortes dicit falsum, que sit a.*"

[5] Bradwardine [2010, ¶6.3]: "*Prima* ⟨*suppositio*⟩ *est ista: quelibet propositio est vera vel falsa.*"

[6] Tim Maudlin [2004] refers to **Tr**-in and **Tr**-out as Upwards and Downwards T-Inference, respectively.

the truth of a proposition that something it signifies obtain. Rather, for it to be true, everything it signifies must be the case. Bradwardine expresses these two principles in his first and second definitions, of truth and falsity:

> "The definitions are two, of which the first is this: A true proposition is an utterance signifying only as things are. The second is this: A false proposition is an utterance signifying other than things are."[7]

We can express these in our preferred notation as follows:

$$(D1) \qquad \mathbf{Tr}(s) := (\exists p)\mathbf{Sig}(s,p) \wedge (\forall p)(\mathbf{Sig}(s,p) \to p)$$

$$(D2) \qquad \mathbf{Fa}(s) := (\exists p)(\mathbf{Sig}(s,p) \wedge \neg p)$$

Given a definition of proposition:

$$(D0) \qquad \mathbf{Prop}(s) := (\exists p)\mathbf{Sig}(s,p)$$

it follows that only propositions can be true or false, and every proposition is one or the other:

$$(P1) \qquad \mathbf{Prop}(s) \to (\mathbf{Tr}(s) \vee \mathbf{Fa}(s))$$

Bradwardine's solution to the paradoxes lies in his weakening of (**Tr**-in), strengthening its right-hand side in his definition, $(D1)$. For a proposition to be true, everything it signifies must obtain. Looking back at the paradoxical reasoning about A, we see that, having proved that A was false, we immediately and unthinkingly inferred by (**Tr**-in) that it is true. However, we have no guarantee that A's being false is *all* that A signifies. If A signifies something else as well, perhaps as a result of the closure principle $(P2)$, then $(D1)$ will not support the final inference that A is true, and paradox will be avoided. That is the rationale of Bradwardine's solution. We do not need to prove that, e.g., A is false. We have a proof of that already. We have too many proofs, both that it is false and that it is true. What is needed is a diagnosis which shows that at least one of these proofs is faulty. Bradwardine's target is the "proof" that A is true.

Bradwardine's first thesis, $(T1)$, is a substitution-thesis:

[7] Bradwardine [2010, ¶6.2]: "*Diffinitiones autem sunt due. Quarum prima est ista: propositio vera est oratio significans tantum sicut est. Secunda est ista: propositio falsa est oratio significans aliter quam est.*"

> "Every proposition whose extreme has many supposita signifies or means affirmation or denial for one of them, and if it has only one, for that."[8]

This relies on the idea that a term has certain *supposita*. The theory of supposition was the medievals' account of reference, a two-level or dualist semantics when coupled to the theory of signification. But we do not need to enter the full intricacies of the theory of supposition to understand Bradwardine's use of Thesis 1. His uses are in general restricted to consideration of the supposition of phrases of the kind 'what s signifies'. We want to distinguish, for example, cases where p is all that s signifies from cases where, say, s signifies not only that p but also, e.g., that q. What that raises for us is a different issue: what is the legitimacy of the propositional quantification used by Fitch and, implicitly, by Bradwardine (and explicitly in my representation of his theses) in formulae like (K) and $(P2)$? How is the quantification over propositions here to be understood? We'll come back to that in § 3.

Before we tackle that question, however, we need to complete our description of the paradox in A, and confirm that Spade was mistaken to attribute (BP) to Bradwardine. The heart of Bradwardine's solution lies in his second thesis, $(T2)$:

> "If some proposition signifies itself not to be true or itself to be false, it signifies itself to be true and is false."[9]

Suppose s signifies itself not to be true. Then either s signifies only that, or something else as well. Take the first option first. Then if s is not true, either it signifies nothing or what it signifies does not obtain (by $D1$). But we have assumed that it signifies that it is not true. So what it signifies does not obtain, that is, it is true. Hence by $(P2)$, s signifies that it is true. In symbols, assuming $\mathbf{Prop}(s)$:

$$\begin{array}{rcll}
\neg\mathbf{Tr}(s) & := & \exists p(\mathbf{Sig}(s,p) \wedge \neg p) & \text{by } (D1)\text{, given } \mathbf{Prop}(s) \\
 & \to & \neg\neg\mathbf{Tr}(s) & \begin{cases}\text{by } (T1)\text{, assuming } \neg\mathbf{Tr}(s) \text{ is all} \\ \text{that } s \text{ signifies}\end{cases} \\
 & \to & \mathbf{Tr}(s) & \text{by Double Negation} \\
\text{But } \mathbf{Sig}(s,\neg\mathbf{Tr}(s)), & \text{so} & \mathbf{Sig}(s,\mathbf{Tr}(s)) & \text{by } (P2)
\end{array}$$

[8] Bradwardine [2010, ¶6.4]: "*Post hec omnia, ordine competenti secuntur due conclusiones quarum prima est ista: quelibet propositio cuius extremum multa habet supposita significat sive denotat affirmationem vel negationem pro aliquo istorum, et si unicum pro isto.*"

[9] Bradwardine [2010, ¶6.4]: "*Secunda* ⟨*conclusio*⟩ *est ista: si aliqua propositio significet se non esse veram vel se esse falsam ipsa significat se esse veram et est falsa.*"

On the other hand, suppose s signifies more than just that it is not true, that is, something else as well, call it q. Then if s is not true, what it signifies does not obtain, as before by $(D1)$, that is, it's not the case that s is not true and q, i.e., either s is true or not-q, so again by $(P2)$, since s signifies that s is not true, s must signify that either s is true or not-q. Now 'either s is true or not-q' and q together imply that s is true. Moreover, s signifies that either s is true or not-q (as just shown) and s signifies that q, so by $(P2)$, s signifies that s is true, as required. In symbols, again assuming $\mathbf{Prop}(s)$:

$$\begin{array}{rcll}
\neg\mathbf{Tr}(s) & := & \exists p(\mathbf{Sig}(s,p) \wedge \neg p) & \text{by } (D1) \\
 & \to & \neg(\neg\mathbf{Tr}(s) \wedge q) & \left\{\begin{array}{l}\text{by } (T1)\text{, assuming} \\ \neg\mathbf{Tr}(s) \wedge q \text{ is all} \\ \text{that } s \text{ signifies}\end{array}\right. \\
 & \to & \mathbf{Tr}(s) \vee \neg q & \text{by (P4)}^{10} \\
\text{But } \mathbf{Sig}(s, \neg\mathbf{Tr}(s)), & \text{so} & \mathbf{Sig}(s, \mathbf{Tr}(s) \vee \neg q) & \text{by } (P2) \\
\text{Now } (\mathbf{Tr}(s) \vee \neg q) \wedge q & \to & \mathbf{Tr}(s) & \text{by } (P5)^{11} \\
\text{But} & & & \\
\mathbf{Sig}(s, \mathbf{Tr}(s) \vee \neg q) \text{ and } \mathbf{Sig}(s,q), & \text{so} & \mathbf{Sig}(s, \mathbf{Tr}(s)) & \text{by } (P2')^{12}
\end{array}$$

More succinctly,

$$\begin{array}{rcll}
\neg\mathbf{Tr}(s) & \to & \mathbf{Tr}(s) \vee \neg q & \text{by } (D1) \text{ and } (T1) \text{ as above} \\
 & \to & q \to \mathbf{Tr}(s) & \\
\text{So } \neg\mathbf{Tr}(s) \wedge q & \to & \mathbf{Tr}(s)) & \text{by Importation} \\
\text{But } \mathbf{Sig}(s, \neg\mathbf{Tr}(s) \wedge q), & \text{so} & \mathbf{Sig}(s, \mathbf{Tr}(s)) & \text{by (P2)}
\end{array}$$

Bradwardine's claim is, then, that every proposition signifying itself not to be true (or to be false) also signifies that it is true. Two things follow immediately. First, any such proposition is false, for something it signifies must fail to obtain, by Contravalence. Secondly, the second leg of the paradox is disabled. That s is false, and so not true, is not enough to show that s is true, even though s signifies that it is not true, for it signifies something else as well. $(D1)$ requires that everything s signifies must obtain for s to be

[10] $(P4)$ is De Morgan's Law, Bradwardine [2010, ¶6.3]: "*Quarta ⟨suppositio⟩ est ista: copulativa et disiunctiva ex partibus sibi contradicentibus invicem contradicunt.*"

[11] $(P5)$ is Disjunctive Syllogism, Bradwardine [2010, ¶6.3]: "*Quinta ⟨suppositio⟩ est ista: ex qualibet disiunctiva cum opposito unius suarum partium reliqua pars infertur.*"

[12] Note that Bradwardine here uses a generalized form of $(P2)$, with multiple premises:

$$(P2') \qquad (\forall p)(\forall q)(\forall r)[(p \wedge q \to r) \to (\mathbf{Sig}(s,p) \wedge \mathbf{Sig}(s,q) \to \mathbf{Sig}(s,r))].$$

Clearly, this generalizes to any finite conjunction of premises. Moreover, $(P2')$ entails the closure of signification under conjunction:

$$(\forall p)(\forall q)[\mathbf{Sig}(s,p) \wedge \mathbf{Sig}(s,q) \to \mathbf{Sig}(s, p \wedge q))].$$

true. But that's impossible (by Contravalence) and so s is simply not true, i.e., false.

Now consider A: Socrates utters a falsehood, which is by hypothesis Socrates' only utterance. Then A signifies that A is false (by $P2$), and so it also signifies that it is true, and is false (by $T2$). However, if Bradwardine had endorsed (BP), this solution would fail. First, note that from (CBP) we can infer that, since we have shown that A signifies that A is true, A is not true only if A is true. That readmits the second leg of the paradox, for having shown that A is not true we can immediately infer that A is true, and paradox has returned. Since Bradwardine never mentions (CBP), it is a mistake to attribute it to him.

But even (BP) must be rejected too. To use (BP) to support ($T2$), Bradwardine would have to show that A is not true only if A is true, to apply (BP) to infer that A signifies that A is true. Once again, having inferred that A is not true, he would be forced to conclude that it was also true, and paradox would be back again.

Bradwardine's diagnosis of the paradox in A depends crucially on Bradwardine's notion of *ut nunc* consequence (and signification). For to express the reasoning, we need the contingent fact that Socrates utters A and only A. A signifies that $(\exists x)(Sx \wedge \neg\mathbf{Tr}(x))$, where $Sx :=$ Socrates utters x. We need to know that $(\forall x)(Sx \leftrightarrow x = A)$ in order to infer that A signifies that $\neg\mathbf{Tr}(A)$:

$$\begin{array}{rcll} (\exists x)(Sx \wedge \neg\mathbf{Tr}(x)) & \rightarrow & (\exists x)(x = A \wedge \neg\mathbf{Tr}(x)) & \text{by } (T1) \\ & \rightarrow & \neg\mathbf{Tr}(A) & \\ \text{But } \mathbf{Sig}(A, (\exists x)(Sx \wedge \neg\mathbf{Tr}(x)), & \text{so} & \mathbf{Sig}(A, \neg\mathbf{Tr}(A)). & \end{array}$$

For this reasoning to go through, the consequence relation expressed by '$\rightarrow$' must allow the use of the contingent premise, $(\forall x)(Sx \leftrightarrow x = A)$. That is what *ut nunc* ("as-of-now" or "as a matter of fact") consequence allows:[13]

> "An as-of-now consequence holds for a determinate time and not always. For example: Every man runs, therefore, Socrates runs. For that inference does not always hold, but holds only while Socrates is a man."

As-of-now signification is closed under as-of-now consequence; absolute signification is closed under absolute, that is, necessary consequence. A sig-

[13] Walter Burley [1955] II 1 1, p. 61: "*Consequentia ut nunc est, quae tenet pro determinato tempore et non semper, ut: Omnis homo currit, ergo Sortes currit; ista enim consequentia non tenet pro omni tempore, sed solum tenet, dum Sortes est homo.*"

nifies as-of-now, or as a matter of fact, that it is not true. So by $(T2)$, $\mathbf{Sig}(A, \mathbf{Tr}(A))$, that is, A also signifies as a matter of fact that it is true, and accordingly, A is false, since not everything it signifies (that it is true, and that it is false) can obtain.

3 Propositional Quantification

Propositional quantification of the sort used in (K), $(D1)$ and so on has long been contentious. One common complaint is that expressions like the right-hand side of $(D1)$ cannot be read, or at least, not without introducing some predicate like 'is true': "for all propositions p, if s signifies that p then ..." what? Simply to conclude "... p" seems incomplete, unfinished, left in the air; "then p is true" completes the sentence, or so it seems. But if so, $(D1)$ is circular. The *definiendum* 'true' appears in the *definiens*, and so it can hardly constitute a definition of truth.[14]

Something strange is afoot here, however, for if the consequent of the right-hand side of $(D1)$ can only be read by adding 'is true', then what replaces 'p' there is not a proposition but a name, or at least, a singular term. Yet that is problematic for the antecedent of that formula, 's signifies that p': 'p' cannot there be replaced by a singular term. This has led some to construe 'that p' as itself a singular term, and 'signifies' as a relation between two objects, the expression (replacing) 's' and its signification 'that p'.[15] This is a grave, but tempting error, one which according to Coffa beset the semantic tradition from Wittgenstein's *Tractatus* to Carnap's *Logical Syntax*.[16] The mistake can be traced back to Frege, in fact. It is to suppose that every expression in a proposition is a name. If language can function only by naming, then semantics is impossible. William Kneale expresses the puzzle in this way: a child asks, 'What is an oculist?', and receives the answer, 'An oculist is an eye doctor'. Kneale remarks (Kneale [1972, p. 227]): "Most of us are so familiar with this wide-spread device for extending knowledge that we fail to notice its peculiarity." The adult offering this clarification cannot be using 'oculist' in its normal sense, else his reply would be merely analytic, says Kneale. He must mean, he suggests, something like 'thing called by the name "oculist"'. But even this is not quite right, at least if one construes '"oculist"' as just a name for 'oculist', an idea Kneale rightly rejects on the next page. The puzzle was famously dramatized by Elizabeth Anscombe:[17]

[14] See, e.g., Haack [1978, pp. 130 ff.]; David [2005, pp. 187-90].

[15] See, e.g., Kneale [1972, p. 235].

[16] Coffa [1991], esp. ch. 17. Whether Bradwardine and his contemporaries committed such an error is an issue for another occasion.

[17] Anscombe [1959, p. 84]. Cf. Anscombe [1956-7].

> "It is impossible to be told anyone's name by being told 'That man's name is "Smith"'; for then his name is named, not used as a name, in that statement, so what I hear is the name of his name and not his name."

The function of quotation-marks is not to name an expression, but to display it. If language consisted solely of names, one could not talk about the meaning of those names. It is the ability of language to display the meaning of its terms which permits us to do so. Indeed, Karel Reach, from whom Anscombe drew this puzzle, demonstrates this dramatically in presenting a table, or as he preferred to say, a "museum", to explain his point:[18]

;	Semicolon	Secol
Semicolon	Secol	Sco

What appears in the top row are not names, or at least, are not used as names. The names in the bottom row label the objects in the top row, as labels in a museum-case. Only in this way can we correlate names with the objects which they name.

In his 'Intellectual Autobiography', Carnap tells a charming anecdote about a meeting with Tarski in a café in Vienna (Carnap [1963] p. 60). Carnap asked Tarski about his definition of truth:

> "I assumed that he had in mind a syntactical definition of logical truth or provability. I was surprised when he said that he meant truth in the customary sense, including contingent factual truth. Since I was thinking only in terms of a syntactical metalanguage, I wondered how it was possible to state the truth-condition for a simple sentence like 'this table is black.' Tarski replied, 'This is simple: the sentence "this table is black" is true if and only if this table is black.'"

Coffa is so taken with this story that he tells it twice in his book.[19] Indeed, perhaps his book should have been subtitled 'To the Vienna Café', rather than 'To the Vienna Station', though that would have missed the literary allusion to Edmund Wilson's *To the Finland Station* (Wilson [2003]). Tarski's insight into the definition of truth is portrayed as the "fundamental breakthrough"[20] in the semantic tradition, playing the same role there as Lenin's

[18] Reach [1938, p. 99]. See also Read [1997].
[19] Coffa [1991, pp. 304, 372].
[20] Wilson [2003, 'Introduction' p. xxi].

arrival at St Petersburg in 1918 did for the communist revolution. Carnap came so close, but could not see the final step because he thought talk about meaning was impossible, a legacy of the saying/showing distinction of the *Tractatus*. But, Coffa argues, Carnap had all he needed ready to hand in his metamathematics, or "logical syntax", in particular, the presence in the metalanguage of translations of the object language. Tarski's material adequacy condition has the form:

(T-scheme) $\qquad\qquad s$ is true if and only if p

where what replaces 'p' is a translation into the metalanguage of the object language proposition whose (structural-descriptive) name replaces 's'. In (T-scheme), there is no similar temptation to construe 'p' as a variable whose replacements are singular terms, or to repeat 'is true' after it. As Davidson suggests,[21] (T-scheme) has the same form as 's means that p', or Bradwardine's 's says that p' (and 'a knows that p'). The expression 'says that' is a "connecticate",[22] acting as a connective at one end (taking a propositional variable, 'p') and as a predicate at the other (taking a name variable, 's'). In fact, the grammar of 's says that p' is that it consists of a main clause ('s says') conjoined by the relative conjunction 'that' to a subordinate clause ('p'). To construe 'that p' as a unity, as Kneale, Slater and many others do,[23] is both grammatically and logically wrong, and metaphysically dangerous, for one is thereupon tempted to cast around for a referent for this supposed singular term.

How, then, is the right-hand side of ($D1$) to be read? It's a formula, and as such gives the form of certain expressions, namely, those which result from replacing 's' and 'p' in '$\mathbf{Sig}(s, p) \to p$' by expressions of the appropriate type. Church's theory of types, or Ajdukiewicz's theory of syntactic categories are useful aides to keep these types distinct.[24] Church's type-theoretic notation is more elegant; Ajdukiewicz's more intuitive. Ajdukiewicz takes two basic categories, n, the type of names (or in Church's terminology, individuals) and s, the type of sentences (or for Church, propositions). Given any two categories, a and b, $\frac{\mathsf{a}}{\mathsf{b}}$ is a further category, the category of expressions which take an expression of category b and form an expression of category a. Thus 'says that' is an expression of category $\frac{\mathsf{s}}{\mathsf{ns}}$ $(= \frac{(\frac{\mathsf{s}}{\mathsf{n}})}{\mathsf{s}})$[25] taking a name and a sentence to form a new sentence. 's says that' is a one-place connective (of type $\frac{\mathsf{s}}{\mathsf{s}}$) and 'says that p' is a one-place predicate (of type $\frac{\mathsf{s}}{\mathsf{n}}$).

But the right-hand side of ($D1$) contains a quantifier, '$(\forall p)$', and that

[21] Davidson [1994, p. 23].

[22] The neologism is generally credited to Prior, though only the idea, not the word, occurs in Prior [1971, pp. 19, 135].

[23] Kneale [1972, p. 235]; Slater [2004, p. 60].

[24] Church [1940]; Ajdukiewicz [1967].

[25] Actually, in full, it's $(\frac{\mathsf{s}}{\mathsf{ss}})(\frac{\mathsf{s}}{\mathsf{n}})(\frac{\mathsf{s}}{\mathsf{s}})$.

prompts the question what is the domain of quantification; what does the quantifier range over? It's tempting to read that question as asking what the expressions which replace 'p' in '$\mathbf{Sig}(s, p) \to p$' refer to. That, however, is a mistake, as we have seen, for not all expressions are names or are intended to refer. But it may be answered more charitably by offering a semantics for expressions of the type 's says that p' and 'a knows that p'. Greg Restall (Restall [2008]) has provided this for the former. Variables like 's' range over so-called declarative objects, variables like 'p' over subsets of worlds.

I take the declarative objects to be interpreted sentences, i.e., for short, propositions. Bradwardine's propositions are not token-sentences, despite medieval logicians being frequently interpreted that way.[26] This was brought home to me by some remarks of Catarina Dutilh-Novaes' (Dutilh Novaes [2005, §2]). That they are not simply token-sentences is clear, for example, from Bradwardine's response to the problem of revenge, which arises repeatedly throughout ch. 7 of his treatise. Take Socrates' utterance above of 'Socrates utters a falsehood'. It follows from Bradwardine's thesis ($T2$) that Socrates utters a falsehood. Bradwardine writes:[27]

> "But if it is true that Socrates utters a falsehood, and Socrates says this, then Socrates utters a truth. The reply is that the second premise is false, because he does not utter the proposition presented by you, and conceded by me, but another ⟨proposition⟩ like it, namely A."

His reason is that Socrates' utterance, A, signifies itself to be both false and true, whereas Bradwardine's own utterance of that proposition (call it B) signifies only that A is false, not that it itself is false or that it is true. Bradwardine does not appeal to the fact that Socrates and he uttered different tokens of the proposition 'Socrates utters a falsehood', but that their utterances, though similar (in form), differ in signification. If they did not differ in signification, they would be identical, as Bradwardine's argument a little later shows. He considers the following objection:[28]

> "But the extremes and the copulas of A and B create entirely the same concepts in thought and in the same order, and so the

[26] See, e.g., Hughes [1982, p. 5].

[27] Bradwardine [2010, ¶7.1.1]: "*Contra: tunc hec est vera: Sortes dicit falsum, et Sortes dicit istam, ergo Sortes dicit verum. Dicendum quod minor est falsa, quia non dicit istam propositionem propositam a te et concessam a me, sed aliam sibi similem, scilicet a.*"

[28] Bradwardine [2010, ¶7.2.4]: "*Contra: extrema et copule a et b faciunt omnino easdem intentiones in conceptu et secundum eundem ordinem, ergo intellectus in simplici apprehensione consimiliter omnino ab utroque movetur. Ergo si eisdem intentionibus tali ordine conceptis, propositio composita erit omnino eadem nec diversa: ergo omnino idem significant. Ergo convertuntur.*"

> mind in simple apprehension is affected by each in entirely the same way. Hence if the same concepts are thought in the same order, the proposition composed will be entirely the same and not diverse; hence they signify entirely the same. Therefore they are equivalent."

Of course, he denies this, for A and B do not create the same thought:[29]

> "... it can indeed be inferred that the primary composition of each in thought is the same, but not everything, because each of them signifies primarily that Socrates utters a falsehood and A signifies secondarily that A is false and that A is true, as was shown."

In sum, A signifies as a matter of fact (since it was Socrates' only utterance) that it itself is both true and false. B signifies only that A is false. So A and B are different propositions. To represent such distinctions, we need to employ a "connecticate", the expression 'says that', or 'signifies that', which takes the name of a proposition, s, and a sentence, p, to form a new sentence: 's says that p'.

4 The Knower Paradox

As we saw in §1, Fitch's paradox is not itself self-contradictory, but contradiction is not far away. What is needed is to add diagonalization, or self-reference. Consider the proposition, 'This proposition is true but not known', call it C. If C were known to be true, then it would be known that C was true and unknown, so by (K') it would be known that C was true and known that C was not known, so by (T), it would be known that C was true and not known that C was true. In symbols:

$$\begin{array}{llr} \mathbf{Kn}(\mathbf{Tr}(C)) & \to \quad \mathbf{Kn}(\mathbf{Tr}(C) \wedge \neg\mathbf{Kn}(\mathbf{Tr}(C))) & \text{by } (D3)\text{—below} \\ & \to \quad \mathbf{Kn}(\mathbf{Tr}(C)) \wedge \mathbf{Kn}\neg\mathbf{Kn}(\mathbf{Tr}(C)) & \text{by } (K') \\ & \to \quad \mathbf{Kn}(\mathbf{Tr}(C)) \wedge \neg\mathbf{Kn}(\mathbf{Tr}(C)) & \text{by } (T) \end{array}$$

That's impossible, so we have a proof that C is not (and never can be)

[29] Bradwardine [2010, ¶ad 7.2.4]: "... *bene posset concludi quod utriusque compositio primaria in conceptu est eadem, non tamen quecumque, quia utraque illarum primo significat quod Sortes dicat falsum et a secundario significat a esse falsum, et a esse verum, ut declaratum est.*"

known.[30]

The Knower Paradox differs from C only in dropping the first conjunct, to read, 'This proposition is unknown'. Bradwardine discusses a variety of epistemic paradoxes in ch. 9 of his treatise on insolubles. One of these is the insoluble 'This proposition is not known by you'. Bradwardine phrases the puzzle in the language of obligations;[31] he also puts it in the affirmative: 'This proposition is unknown to you'. Perhaps it's clearer for present purposes to put it like this: let D be the proposition 'D is not known by you' ($\neg\mathbf{Kn}(\mathbf{Tr}(D))$). Suppose D were known by you. Then it would be true (by (T)) and so not known by you (by **Tr**-out). Contradiction. So D is not known by you, by *reductio*. But that is what D signifies, so D is true (by **Tr**-in). Hence D is not known by you (we've proved that) but moreover D is known by you (by Gödel's rule). Paradox.

Gödel's rule is another modal principle of epistemic logic.[32] It says that if one has proved something, one may infer that one knows it. It is called 'Necessitation' in its alethic modal version: if one has proved something, one may infer that it is necessarily true. However, it is not to Gödel's principle that Bradwardine objects in the above proof. In line with the analysis in §2 above, he objects to the application of (**Tr**-in), inferring from the fact that D is not known (by you) that it is true. For although D signifies that D is not known by you, it may signify more than that, and everything it signifies must obtain in order for D to be true, in accord with $(D1)$.

Indeed, we can show that D also signifies that it is true. For suppose that D is false. Then something it signifies must fail (by $D2$), either that you don't know that D is true or whatever else it signifies (call it q), that is, if q obtains then you know that D is true. By Importation, if D is false and q obtains, then D is known by you to be true. By Antilogism (interchanging and negating one premise and the conclusion[33]), if D is not known by you and q obtains, then D is true. But D signifies that D is not known by you and q, so by $(P2)$, D signifies that D is true. In symbols:

[30] Since 'C' is a name, we cannot write 'One knows that C' (which is ill-formed), but must complete it so that what follows 'knows that' is a sentence, not a name, e.g., 'One know that C is true'.

[31] On the medieval doctrine of obligations, see, e.g., Spade [2008].

[32] It is also a basic principle of provability logic. Cf. Rule IV in Löb [1955, p. 116]: "if the formula A is provable, so is the formula $\widetilde{\mathcal{B}}\ (A)$," i.e., that A is provable. Gödel introduced the rule in his formulation of the modal logic S4 in Gödel [1933].

[33] Aristotle called it *reductio per impossibile*: see *An. Pr.* I 5.

$$
\begin{array}{llcll}
 & \mathbf{Fa}(D) & \to & \exists p(\mathbf{Sig}(D,p) \wedge \neg p) & (D2) \\
 & & \to & \neg\neg\mathbf{Kn}(\mathbf{Tr}(D)) \vee \neg q & (T1) \\
 & & \to & q \to \mathbf{Kn}(\mathbf{Tr}(D)) & \\
\text{So} & \mathbf{Fa}(D) \wedge q & \to & \mathbf{Kn}(\mathbf{Tr}(D)) & \text{Importation} \\
\text{whence} & \neg\mathbf{Kn}(\mathbf{Tr}(D)) \wedge q & \to & \mathbf{Tr}(D) & \text{Antilogism and } (P1) \\
\text{But} & \mathbf{Sig}(D, \neg\mathbf{Kn}(\mathbf{Tr}(D)) \wedge q), & \text{so} & \mathbf{Sig}(D, \mathbf{Tr}(D)) & (P2)
\end{array}
$$

Hence the application of **Tr**-in, or ($D1$), in the proof of paradox, is fallacious. D signifies more than just that you don't know D, and so it does not suffice that you don't know D in order to conclude that D is true.

But this derivation does not, in contrast with earlier insolubles, tell us whether D is true or false. There is nothing obviously contradictory in the fact that D signifies both that you don't know it and that it is true. D turns out to be equivalent to C, and we can see from the earlier proof that D could never be known. Yet some true propositions are unknowable, as Fitch showed. So it seems still to be an open question whether D is true. Bradwardine sets out to show that it is not. First, (K) itself is explicitly endorsed by Bradwardine [2010, ¶9.2]:[34]

> "This inference is valid and known to you to be valid, and the premise is known to you to be true, whence so is the conclusion."

Next, we need a definition of 'knows that'. Then Bradwardine can state his central epistemic thesis, ($T3$):[35]

($D3$) To know a proposition is to know wholly so to be as is signified by it.[36]

($T3$) If some proposition only signifies itself not to be known by someone, or if in addition it only signifies some thing or things known by him, then it signifies that he does not know that he does not know it.

[34] "*Ista consequentia est bona et scita a te esse bona, et antecedens est scitum a te esse verum, ergo et consequens.*" See also, e.g., Boh [1983, p. 75], where the principle is also endorsed by Heytesbury, and p. 97, where Boh recognises that Strode's statement of the stronger principle of the closure of knowledge under consequence is elliptical for the weaker principle. Cf. p. 39.

[35] Bradwardine [2010, ¶9.3]: "*Diffinitio est ista: scire propositionem est scire totaliter sic esse sicut significatur per eam. Conclusio est ista: si aliqua propositio tantum significat se ab aliquo nesciri, vel si cum hoc tantum significet scitum vel scita ab illo, significat nesciri ab illo ipsam ab eodem nesciri.*"

[36] Cf. Nuchelmans [1973, p. 216]: "the object of knowledge is the significate of the *propositio*"; and "what is known is ... that which is signified by the *propositio*, the *totale significatum*." (p. 212)

The proof of ($T3$) follows the same lines as the proof of ($T2$). Here is a compact version of the proof, like that I gave of ($T2$) in §2. Suppose s signifies that it is not known (by someone), and anything else it signifies is known by him: in symbols: $\mathbf{Sig}(s, \neg\mathbf{Kn}(\mathbf{Tr}(s)) \wedge q) \wedge \mathbf{Kn}(q)$. Then:

$$\begin{array}{lrcll}
 & \neg\mathbf{Kn}(\mathbf{Tr}(s)) \wedge q & \to & \neg\mathbf{Kn}(\neg\mathbf{Kn}(\mathbf{Tr}(s)) \wedge q) & (D3 \text{ and } T1) \\
 & & \to & \neg(\mathbf{Kn}\neg\mathbf{Kn}(\mathbf{Tr}(s)) \wedge \mathbf{Kn}(q)) & (\text{by } K') \\
 & & \to & \neg\mathbf{Kn}\neg\mathbf{Kn}(\mathbf{Tr}(s)) \vee \neg\mathbf{Kn}(q) & (P4) \\
\text{But} & (\neg\mathbf{Kn}\neg\mathbf{Kn}(\mathbf{Tr}(s)) \vee \neg\mathbf{Kn}(q)) \wedge \mathbf{Kn}(q) & \to & \neg\mathbf{Kn}\neg\mathbf{Kn}(\mathbf{Tr}(s)) & (P5) \\
\text{and} & \mathbf{Kn}(q).\ \text{So}\ \neg\mathbf{Kn}(\mathbf{Tr}(s)) \wedge q & \to & \neg\mathbf{Kn}\neg\mathbf{Kn}(\mathbf{Tr}(s)) & \\
\text{But} & \mathbf{Sig}(s, \neg\mathbf{Kn}(\mathbf{Tr}(s)) \wedge q), & \text{so} & \mathbf{Sig}(s, \neg\mathbf{Kn}\neg\mathbf{Kn}(\mathbf{Tr}(s))) & (P2)
\end{array}$$

Thus s signifies that it is not known that s is not known. But suppose s were known. Then it would be true, by (T), and so not known, by ($D1$). Hence by *reductio*, s is not known, whence it is known that s is not known, by Gödel's rule, since we've proved that s is not known. So something s signifies fails to obtain, whence by ($D2$), s is false.

What, however, of Bradwardine's own insoluble, D: 'This proposition is unknown to you', in ¶9.2? It is this paradox which he uses to motivate thesis ($T3$). But his subsequent diagnosis is unclear. The resolution seems straightforward enough. D signifies (at least) that D is not known to you. Assuming that anything else D signifies is known to you, it follows by ($T3$) that D signifies that you don't know that you don't know D. But you do know that, for Bradwardine accepts the reasoning by (T), ($D1$) and *reductio* that D is false, and so (by T) cannot be known. Hence by Gödel's rule, it is known that it is unknown. So something D signifies fails to obtain, so by ($D2$), D is simply false.

Bradwardine agrees: he infers by ($T3$) that D "signifies that it is unknown to Socrates that [D] is unknown to Socrates, which is false, because it is known to Socrates that [D] is unknown to Socrates."[37] He then tries to justify this "assumption", but proceeds to invoke ($T3$), obscurely, to show once again that D signifies that it is unknown to Socrates that D is unknown to Socrates,[38] whereas what needs to be shown is that Socrates does know that he does not know D.

Moreover, there is actually something rather puzzling about the proof of ($T3$). For it is an hypothesis of the proof that whatever s signifies besides

[37] Bradwardine [2010, ¶ad 9.2]: "*hoc significat [d] nesciri a Sorte nesciri a Sorte, quod falsum est, quia [d] nesciri a Sorte scitur a Sorte.*"

[38] "*. . . quia vel [d] significat [d] nesciri a Sorte nesciri a Sorte vel non: si sic, habetur intentum; si non, tunc non potest assignari aliquod nescitum a Sorte quod significatur per [d], ergo [d] significat se nesciri a Sorte, et cum hoc: tantum scitum vel scita, scilicet: aliquod propositum nesciri a Sorte et alia vera sequentia. Ergo per secundam partem conclusionis predicte, [d] significat [d] nesciri a Sorte vel a te nesciri a Sorte vel a te. Et hoc est quod probare volumus.*"

s's being unknown is itself known. The proof proceeds to show that s also signifies that it is not known that s is not known. But that is false, for it is known that s is not known. Hence something s signifies is false, and so (by T) it is not known.

One response is that the proof shows that nothing can satisfy its hypothesis, just as with the first part of the proof of ($T2$). There Bradwardine showed that if s signifies only that s is not true then it also signifies that s is true. So no proposition can signify only that it itself is not true. In two MSS, there is an additional, thirteenth chapter, which declares it was composed subsequent to the main treatise, and may or may not be by Bradwardine. It certainly reads like a series of later glosses by Bradwardine himself. Bradwardine [2010, ¶A.4.3][39] considers the objection that since words have signification by convention and imposition, we can just lay down that some proposition signifies only that it itself is not true, and then paradox returns. The author replies just as one would expect Bradwardine to do, invoking ($T2$): that thesis shows that one cannot do that. If one produces, by convention, a proposition which signifies that it itself is not true, then one can prove that that is not all it signifies.

But in proving ($T2$), Bradwardine proceeded from the case where s signified only that s is not true to that where it signifies more, and showed that in that case too it signifies that s is true. For the first step of the proof shows that if s signifies only that s is not true then it also signifies that s is true. So s cannot signify only that s is not true. The second part of the proof is essential, showing that if s signifies that it is not true and more besides, as it must, s signifies that it is true. In his proof of ($T3$), he does not move onto the second part of the proof. If a proposition signifies only that it is unknown and other things, all of which are known (and so true), then it also signifies things that are false and so not known. So by *reductio*, it cannot signify only that it is unknown and other things that are known. But there appears to be a gap in showing that it also signifies that it is not known that it is not known. Bradwardine has yet to show us that if it signifies that it is not known and other things, some of which are not known, then it signifies that it is not known that it is not known. Bradwardine does not do that, and it is not clear how it could be done.

[39]Or at least, the author of the incidental chapter. This passage in fact also occurs in ch. 7 of the main treatise in one manuscript.

5 Conclusion

Fitch's paradox appears to show that if not everything is known, then not everything is knowable. Yet the Principle of Knowability, that whatever is true is knowable, seems attractive, both to a constructivist, who thinks that unknowable truth is unintelligible, and to a realist, who thinks that there should be no bar to our epistemic powers. What Fitch observed was that though one knows that there are unknown truths, one could not know of any proposition that it was both true and unknown. Indeed, one can prove of any proposition which says of itself that it is true and unknown, that it is both true and unknown. But that is a contradiction, for if we can prove it is true, then we know it is true, and so it cannot be unknown, and so is not true.

In his discussion of "insolubles" in the fourteenth century, Thomas Bradwardine considered epistemic puzzles similar to this, in particular, the Knower paradox, a puzzle centring on the proposition which says of itself that it is not known. Bradwardine's solution to the insolubles depends on a closure principle for signification, that any proposition signifies whatever follows from what it signifies. D, which says of itself that it is unknown, also signifies that it is true, and so is equivalent to proposition C, which says of itself that it is an unknown truth. Bradwardine's diagnosis of the Knower paradox is that any proposition which signifies that it is unknown also signifies that it is unknown that it is unknown, provided whatever else it signifies is known. So the Knower paradox signifies not only that it is unknown, but also that it is not known that it is not known. Since it is known that it is unknown, it is false, and so not known.

So too with the equivalent proposition which says of itself that it is an unknown truth. It too signifies that it is unknown that it is unknown, so false, since it is so known. It is no counter-example to the Principle of Knowability, for it is not true.

Nonetheless, there is a lacuna in Bradwardine's argument, for his proof that such propositions which say of themselves that they are unknown also signify that it is unknown that they are unknown, is incomplete. Bradwardine proves it only, as indicated, for such propositions for which what else they signify is known. But that it is not known that they are not known is not known—it cannot be, for it is false: it is known that they are unknown.

If this lacuna can be filled, we have an answer to our paradox. Take C, which signifies that it itself is true and unknown. Then C is equivalent to D, which signifies that it is unknown, for we can show that D also signifies that it is true. Assuming D satisfies the hypothesis of $(T3)$, it also signifies that it is not known that D is not known. Yet that is not so. It is known that D is not known. So D is false. Hence C too is false, since it is equivalent to D.

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