Consequence and Signification in Fourteenth-Century Logic

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Abstract

Forty years ago, Niels Green-Pedersen listed five different accounts of valid consequence, variously promoted by logicians in the early fourteenth century and discussed by Niels Drukken of Denmark in his commentary on Aristotle's *Prior Analytics*, written in Paris in the late 1330s. Two of these arguably fail to give defining conditions: truth preservation was shown by Buridan and others to be neither necessary nor sufficient; incompatibility of the opposite of the conclusion with the premises is merely circular if incompatibility is analysed in terms of consequence. Buridan was perhaps the first to define consequence in terms of preservation of what we might dub verification, that is, signifying as things are. John Mair pinpointed a sophism which threatens to undermine this proposal. Bradwardine turned it around: he suggested that a necessary condition on consequence was that the premises signify everything the conclusion signifies. Dumbleton gave counterexamples to Bradwardine's postulates in which the conclusion arguably signifies more than, or even completely differently from the premises. Yet a long-standing tradition held that some species of validity depend on the conclusion being in some way contained in the premises. We explore the connection between signification and consequence and its role in solving the insolubles.

1 Definitions of Valid Consequence

Niels Green-Pedersen (1981, pp.53-58) finds the following varied accounts of valid inference, or consequence, in Niels Drukken of Denmark's *Questions on Aristotle's 'Prior Analytics'* (edited by Green-Pedersen in Green-Pedersen and Ebbesen, 1997), written in Paris in the late 1330s: a consecution[1] is valid iff

1. the premises cannot be true without the conclusion being true, or

2. it is impossible for things to be as the premises signify without their being as the conclusion signifies, or

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[1] Ciola (2018, p.274) notes the ambiguity in many medieval treatises between *consequentia* understood, on the one hand, as the linguistic item (*oratio*) consisting of premises (*antecedens*) and conclusion (*consequens*) and, on the other, understood as the relation between these two items. I shall use 'consequence' or 'valid inference' to translate *consequentia* when it is the defining property or relation of validity that is in question, and 'consecution' when a particular sequence of premises and conclusion, valid or not, is being discussed.

3. the contradictory opposite of the conclusion is incompatible with the premises;

and it is formally valid iff

4. anything which is signified by the conclusion is signified by the premises (that is, the conclusion is contained or understood in the premises), or

5. it holds by its form, that is, for any uniform substitution of terms.

All five were espoused by various contemporaries of Drukken's, and each was rejected by others.

1.1 Truth-Preservation

Does validity consist in preservation of truth from premises to conclusion, that is, that the premises cannot be true unless the conclusion is also true? John Buridan, also writing in Paris around the same time as Drukken, pointed out that on this account, any consecution would be invalid simply by not formulating the conclusion. For sentences, thought of by the medievals as concrete particulars, can only be true or false if they are actually formulated. So we need at least to add "when they are formed together" to the definition. But, Buridan argued:

> "... this definition is even now not good, because:
>   No sentence is negative, so no ass is running,
> is not a sound[2] consecution ... because the opposite of the premise does not follow from the opposite of the conclusion, that is, this does not follow:
>   Some ass is running, so some sentence is negative.
> But according to the definition as amended one must concede that it is sound, since it is impossible for the premise to be true, so it is impossible for it to be true the conclusion not being true."[3]

Drukken has his own counterexample:[4]

> "Supposing that there are many sentences and each of them is affirmative (which is possible), then one argues like this: 'Each sentence is affirmative, therefore none is negative'. The consecution is formally valid, and the premise is true by

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[2]I shall use 'sound' to translate *bona* as applied to consecutions and 'is valid' to translate *valet*, though the medievals appear to have treated them as equivalent.

[3]This is a somewhat free translation of the Latin, reordering the argument for the sake of clarity (cf. John Buridan, 2014, I 3, p.67): *Sed adhuc dico quod haec descriptio non est bona, quia hic non est bona consequentia: «nulla propositio est negatiua; ergo nullus asinus currit», et tamen secundum dictam descriptionem oporteret eam concedere esse bonam; ergo etc. Primam praemissam probo. Quia ex opposito consequentis non sequitur oppositum antecedentis; non enim sequitur: «quidam asinus currit; ergo quaedam propositio est negatiua». Secunda autem praemissa manifesta est. Quia primam, scilicet quae designatur esse antecedens, impossibile est esse ueram; ergo impossibile est ipsam esse ueram alia non existente uera.* (John Buridan, 1976, pp.21-22) As Ciola (2018, pp.277-8) notes, Albert of Saxony and Marsilius of Inghen present similar counterexamples.

[4]It is also found in Pseudo-Scotus (2001, p.228) and repeated some dozen or so years later by Buridan in his *Sophismata*: John Buridan (2004, cap.8: Primum Sophisma, p.140), Eng.tr. in John Buridan (2001, p. 952).

the scenario proposed, and the conclusion is false. Therefore a falsehood follows from a true sentence in a formally valid consecution. The premise is true by the scenario, and the conclusion is false, which is proved because this sentence, 'No sentence is negative', is simply false because it itself is negative, therefore, some sentence is negative. Therefore 'No sentence is negative' is false, and this was the conclusion. Therefore the conclusion was false."[5]

Thus truth-preservation is not sufficient for validity and the account of validity must be revised.

Roger Swyneshed, writing in Oxford a few years earlier, gave a different kind of counterexample, since he incorporated the exclusion of self-falsifying sentences into his account of truth in order to solve the insolubles (that is, the logical paradoxes):[6]

"A true sentence is a sentence not falsifying itself signifying principally as things are either naturally or by imposition whereby it was last imposed to signify ... A false sentence is an utterance falsifying itself or an utterance not falsifying itself signifying principally other than things are either naturally or by imposition whereby it was last imposed to signify."[7]

He then argues for three iconoclastic theses:

1. There is a false sentence signifying principally as things are

2. There is a formal and valid consecution in which the false follows from the true

3. Two contradictories mutually contradicting one another are both false.

The examples he gives illustrating and demonstrating all three theses are based on the insoluble 'This sentence is false'. In particular, the second thesis is proved by considering the consecution:

The conclusion of this consecution is false,
so the conclusion of this consecution is false.

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[5]See the fifth argument in response to Question 7 (Green-Pedersen and Ebbesen, 1997, ll.1-2, p.32): "whether in any possible scenario a false conclusion follows by a formal consequence from true premises" (*utrum aliquo casu possibili posito ex antecedente vero formali consequentia sequitur consequens falsum*): *Item quinto: Supponendo quod sint multae propositiones, et quod quaelibet illarum sit affirmativa—hoc est possibile—tunc arguitur sic: 'Quaelibet propositio est affirmativa. Ergo nulla est negativa'. Consequentia est formalis, et antecedens verum per casum positum, et consequens falsum. Ergo formali consequentia ex vera sequitur falsum. Antecedens verum per casum, et consequens est falsum. Quod probatur, quia ista propositio est simpliciter falsa: 'Nulla propositio est negativa', quia ipsamet est negativa. Ergo aliqua propositio est negativa. Haec ergo est falsa: 'Nulla est negativa'. Et haec fuit consequens. Ergo consequens fuit falsum.* (1997, ll.50-59, pp.33-34)

[6]On the medieval theories of insolubles, see, e.g., Spade and Read (2021).

[7] Spade (1979, §§18-19, pp.185-6): *... propositio vera est propositio non falsificans se principaliter sicut est significans naturaliter aut ex impositione vel impositionibus qua vel quibus ultimo fuit imposita ad significandum ... propositio falsa est oratio falsificans se vel oratio non falsificans se principaliter aliter quam est significans naturaliter, ex impositione, vel impositionibus qua vel quibus ultimo fuit imposita ad significandum.*

The conclusion is false because it falsifies itself. So the premise is true since it truly says of the conclusion that it is false—and the premise does not refer to, or falsify, itself. Thus truth-preservation is not necessary for validity—at least, not for Swyneshed. So what is the correct account of validity?

1.2 Verification-preservation

Paul Spade (1983, p.113 n.32) conjectured that for Swyneshed, validity was preservation of signifying as things are, which we might dub "verification-preservation". Not only is the premise "verified", so too is the conclusion, since it is false, which is what it signifies, for it is false because it falsifies itself.[8] This account would certainly validate the consecution above.[9]

So does validity consist in preservation of verification, that is, of signifying as things are, from premises to conclusion? That is, does validity mean that the premises cannot signify as things are unless the conclusion does so too?[10] John Mair, writing in Paris in the early sixteenth century, presented a counterexample to this proposal:

> The conclusion of this consecution signifies other than things are
> So the conclusion of this consecution signifies other than things are,

or in brief:

> The conclusion of this consecution is not verified
> So the conclusion of this consecution is not verified.

The consecution would seem to be valid, since premise and conclusion say the same thing. But the consecution does not preserve verification, as Mair argues:

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[8]One might be tempted to use in place of 'verified' and 'verifcation' the terms 'firm' and 'firmness', which Spade introduced in his translation and study of Heytesbury's *Insolubilia* (William Heytesbury, 1979, p.81). Spade (1983) was published after William Heytesbury (1979). But Spade doesn't mention firmness in the 1983 paper, and rightly so, since firmness is not signifying as things are (or as is the case), but is "its being the case as the sentence commonly pretends to signify" (Spade's translation of *ita est sicut communiter pretendunt*), that is, things' being as it customarily, or principally, or exactly signifies. For Swyneshed, this is just what it signifies, since he rejects any additional or hidden signification, but it wouldn't work for Buridan (at least, not in his *Treatise on Consequences*, where he conjectures that insolubles have an additional signification of their own truth) or Bradwardine or Heytesbury. Note that Buridan says (2014, I 3) that "one proposition is antecedent to another which is so related to it that it is impossible for things to be altogether as it signifies unless they are altogether as the other signifies when they are proposed together" (*illa propositio est antecedens ad aliam quae sic se habet ad illam quod impossibile est qualitercumque ipsa significat sic esse quin qualitercumque illa alia significat sic sit ipsis simul propositis*), so presumably including the additional signification of their own truth for premises and conclusion when they are insoluble. So it would be misleading to use 'correspondence' in place of 'verification' (and 'corresponds' in place of 'is verified'), for Hughes (in his 'Introduction' to John Buridan 1982, p.17) explicitly contrasts his "correspondence-conditions" with the full truth-condition for a sentence. See also Spade's comment in (William Heytesbury, 1979, 93 n.51).

[9]I argued in Read (2020, §4) that Swyneshed's account of validity could not be verification-preservation (which I mistakenly called firmness-preservation), although I had to leave it an open question, pending further reseaarch, what account he would, or did give.

[10]Verification-preservation is the account offered by John Buridan (2014, I 3) and Albert of Saxony (Albert von Sachsen) (2010, IV 1). Thus Buridan may well have been the first, writing just a few years after Swyneshed, to propose verification-preservation as the correct account of validity.

> "This consecution is sound, 'This conclusion signifies other than things are, therefore, this conclusion signifies other than things are', referring to the conclusion each time. For it proceeds from one synonym to another, but the conclusion signifies other than things are and the premise as things are. Proof: because either the conclusion signifies as things are or other than things are. If the second, we are done; if the first, and it signifies that the conclusion signifies other than things are, and so it is (you agree), therefore the conclusion signifies other than things are. From this it is clear that the premise signifies as things are, so [the definition is bad]."[11]

The conclusion signifies other than things are because, if it signified as things are, it would signify other than things are, since that is what it signifies. Thus we have a valid consecution whose premise is verified and conclusion unverified. Hence, the correct account of validity cannot be verification-preservation.

1.3 Incompatibility

Paul of Venice, among others,[12] had a different account of validity. Paul spent the years 1390-93 in Oxford, and wrote in his *Logica Parva* when he returned to Italy:

> "A sound consecution is one in which the opposite of the conclusion is incompatible with the premises."[13]

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[11]Johannis Mair (1527, f.142rb): *Secundo argumentor, hec consequentia est bona, hoc consequens significat aliter esse quam est, ergo hoc consequens significat aliter esse quam est, demonstrando consequens utrobique. Proceditur enim a synonimo ad synonimum, et tamen consequens significat aliter esse quam est, et antecedens sicut est. Probatio quia vel consequens significat taliter sicut est vel aliter esse quam est. Si secundum intentum habetur, si prius et ipsum significat consequens significare aliter esse quam est, et ita est per te, ergo consequens significat aliter esse quam est. Et ex illo patet quod antecedens significat taliter esse qualiter est in re, igitur.*

[12]See, e.g., John of Wesel (1996, §56): "That this consecution is sound is proved: the opposite of the conclusion is contradictorily inconsistent with the premise" (*Quod ista consequentia sit bona probatur: Oppositum consequentis repugnat contradictorie antecedenti*); Peter of Mantua, *Logica*, cited in Pozzi (1978, 17, 281): "A consecution indicated by 'if' or 'therefore' is a necessary relation of two sentences where the contradictory of the second cannot stand with the first without new imposition [of meaning] or can be convertible with one such without new imposition" (*Consequentia bona denominata a li 'si' vel 'ergo' est necessaria habitudo duarum propositionum quarum secundae non potest contradictorium stare cum prima sine nova impositione vel potest esse convertibilis cum una tali sine nova impositione*).

[13]Paulus Venetus (2002, ch.III §1, p.52): *Consequentia bona est illa cuius oppositum consequentis repugnat antecedenti.* He repeats this in the *Logica Magna*: both in the 'Consequences' (see Paulus Venetus, 1990, p.80), where Hughes clearly takes this to be a definition: "A valid inference which signifies in accordance with the composition of its elements may be defined as one in which the contradictory of its conclusion would be incompatible with the premiss of that inference, given that these signify as they do; and by 'as they do' I refer to what they customarily signify" (*Consequentia bona significans iuxta compositionem suarum partium dicitur esse illa cuius contradictorium consequentis potest repugnare antecedenti eiusdem, ipsis sic significantibus; et demonstro per ly .sic. ipsorum significata consueta*); and in the 'Insolubles' (see Paul of Venice, 2022, §2.2.5): "For, in agreement with everybody, I mean that if there is [a consecution] that signifies by the composition of its parts and the opposite of the conclusion is formally incompatible with the premise, that [consecution] is formally sound" (*Volo namque concordanter cum omnibus quod si est aliqua consequentia significans ex compositione suarum partium, et oppositum consequentis repugnat formaliter antecedenti, quod illa consequentia sit bona et formalis*).

Of course, whether this account deals with the counterexamples above depends crucially on the account of incompatibility. Paul writes in his *Logica Magna*:

> "Two sentences are mutually incompatible when, signifying as they do, they cannot be nor can have been nor could be true together, or at least their significates cannot be nor can have been nor could be true together."[14]

In the same vein, Gerard Odo, writing in Paris in the 1320s, noted:

> "In any consecution the opposite of the conclusion cannot stand with the premises,"[15]

that is, they can't both be true or at least they can't both be verified, and they cannot stand together—presumably, on pain of contradiction.[16]

First, take Buridan's counterexample to truth-preservation: no sentence being negative is compatible with an ass running, so the consecution is correctly deemed to be invalid. More surprisingly, Swyneshed's counterexample satisfies the "incompatibility"-criterion: for 'The conclusion of this consecution is not false' clearly cannot stand with 'The conclusion of this consecution is false' without contradiction. Finally, Mair's consecution also satisfies the "incompatibility"- criterion of validity: that is, 'The conclusion of this consecution does not signify other than things are' evidently cannot stand with 'The conclusion of this consecution signifies other than things are' without contradiction.

However, one might worry that there is a circularity in the definition. For whether things can stand together without contradiction would seem to mean "their standing together (or their both signifying as things are) does not entail a contradiction". But 'entail' is just another word for (the consecution being) 'valid'. Indeed, for Burley, for example, the incompatibility is not the defining property, but a useful mark:

> "The fourth main rule: Whatever is compatible with the premise is compatible with the conclusion ... The third rule that follows is that in every sound inference the opposition of the conclusion is incompatible with the premises."[17]

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[14]Paulus Venetus (1990, p.21): ... *ad hoc quod duae propositiones ad invicem repugnent requiritur quod ipsae, sic significando, non possint nec poteurint nec poterunt esse simul verae, vel saltem quod significata illarum non possint nec potuerint nec poterunt esse simul vera.*

[15]Giraldus Odonis (1997, p.337): *In quacumque consequentia oppositum consequentis non potest stare cum antecedente.*

[16]Interestingly, Ralph Strode (Seaton, 1973, §1.2.63) distinguishes being incompatible (*repugnans*) from the impossibility of standing together: "But although the contradictory of the conclusion of a sound consecution never stands with its premises, it is not, however, necessary that the opposite of the conclusion is always incompatible with the premises" (*Sed licet numquam contraditorium consequentis consequentie bone stet cum suo antecedente, non tamen oportet quod semper oppositum consequentis repugnet antecedenti*). The example he gives of sentences which cannot stand together but are not incompatible are 'You are not white' and 'Nothing exists' (*Multe enim propositiones non stant simul que tamen non repugnant, sicut iste: 'tu non es albus' et 'nihil est'. Repugnantia namque dicuntur ex quorum uno formaliter sequitur contradictorium alterius, ut 'tu sedes' et 'tu stas'*).

[17]Walter Burley (1955, 63)(2000, 149 §275): *IV^a Regula principalis: Quicquid stat cum antecedente, stat cum consequente ... Tertia regula, quae sequitur, est, quod in omni consequentia bona oppositum consequentis repugnat antecedenti.*

Indeed, in the same vein, many authors say that a consecution is valid just if the opposite of the premises follows from the opposite of the conclusion But clearly, this cannot be a definition, otherwise we would have a regress.[18]

Drukken's fifth definition similarly presupposes a prior definition of validity:

> "Some say that a formal consecution is one which holds thanks to the form of the combination of the terms, so that wherever there is a similar combination of terms, however those terms signify, there is always a sound consecution in the same way."[19]

Tarski used a similar account to give a reductive definition of consequence without recourse to modal notions (like necessity or incompatibility). However, the medievals, notably Parisians like Buridan, used the term-invariance test to distinguish formal validity from validity more generally, and material validity in particular.[20] Hence Drukken's fifth definition presupposes a prior definition of validity itself.

1.4 Containment

Drukken's fourth criterion similarly sets out to define "formal validity", but without the dependence on a prior definition of validity:

> "In every formal consecution the conclusion should be included in the premises so understood that the whole of what is signified by the conclusion should be signified by the premises ... [that is,] in every sound consecution what is signified by the premises and the conclusion is the same, so that whatever is signified by the conclusion should be signified by the premises."[21]

This account of validity is particularly associated with the English school of logic in the fourteenth century.[22] Niels Drukken seems to endorse this account himself. Unsurprisingly, he was a member of the so-called "English Nation" at the University of Paris—though he himself was from Denmark (Dacia). A consecution is not formally valid, he says, if the

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[18]Richard Ferribridge presents a counterexample to the incompatibility criterion in his *Consequentiae* (Pozzi, 1978, 262-71): consider the consecution '*A* will signify precisely that everything true will be false, therefore *A* will be false' (*a significa⟨bi⟩t praecise quod quodlibet verum erit falsum, ergo a erit falsum*) . Ferrybridge argues that the opposite of the conclusion is incompatible with the premise, but the consecution is invalid, for *A* could continue to signify that everything true will be false and still be true since everything true will be false. The example was later used by Paul of Venice in his *Quadratura* to motivate his solution there and in his *Logica Parva* to the insolubles: see Read (2022).

[19]Green-Pedersen and Ebbesen (1997, Q.14 ll.110-13, p.81): *Quidam dicunt quod formalis consequentia est talis quae tenet gratia formae complexionis terminorum, ita quod ubicumque erit consimilis complexio terminorum, qualitercumque illi termini significent, semper eodem modo, est consequentia bona.*

[20]See, e.g., John Buridan (2014, 'Introduction' §3.1). However, we will see in §2.3 that Drukken rejects the notion of material validity, so that his fifth criterion follows from his fourth.

[21]Green-Pedersen and Ebbesen (1997, Q.4 ll.82-4, Q.9 ll.171-3, pp.21,50)' *... in omni consequentia formali consequens debet includi in antecedente ad istum intellectum quod totum significatum consequentis debet esse significatum antecedentis ... in omni bona consequentia significatum antecedentis et consequentis sunt idem, ita quod quicquid significatur per consequens debet significari per antecedens.*

[22]See, e.g., Dutilh Novaes (2020, §3.1) and Ashworth and Spade (1992, p.39). It is dubbed the "containment" condition in Martin (1987, pp.392-3).

conclusion signifies more than the premises. Whatever the conclusion signifies must already have been signified in the premises.[23]

As Green-Pedersen (1981, pp.35-6) notes, Drukken considers it impossible to prove the rule that in a formally valid consecution the premise can't be true and the conclusion at the same time false. Nonetheless, he proceeds to argue for it.[24] His argument seems to turn on the claim that there must be some connection between what the premises signify and what the conclusion signifies. For an understanding of what the premises signify naturally leads to inferring the conclusion. Indeed, if there were some part of the signification of the conclusion which exceeded that of the premises, inferring that part from the premises would be just as unsound. So the whole of what the conclusion signifies must be contained in the premises:

> "Every sound consecution is sound on the basis of what is signified by the premises and what is signified by the conclusion. Then either what is signified by the conclusion is the same as what is signified by the premises or they are disparate in some respect. If they are disparate in some respect, then the intellect would never infer the conclusion from the premises any more than it would infer, 'A man is running, therefore a stick is standing in the corner'. For it's called a consequence [i.e., valid consecution] when the understanding from its natural judgment infers the conclusion from the premises and from the understanding of the premises it understands what the conclusion signifies. If it is said that what the conclusion signifies is the same as what the premises signify, then either it is wholly the same or in some way it is and in some way not. If in some way it is and in some way not, imagine dividing it into what exceeds [the premises] and what doesn't. Then because of what exceeds the premises they do not imply the conclusion, because what exceeds is wholly disparate from what is signified by the premises. So it is necessary that the whole of what is signified by the conclusion is signified by the premises. Therefore, if the premises are true, the conclusion is true, because what they signify is entirely the same; for that reason if one sentence is true, so is the other. This is what earlier writers said, that in every sound consecution the conclusion is contained in the premises, that is, if it is formally sound, the whole of what is signified by the conclusion is signified by the premises, but not [necessarily] vice versa."[25]

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[23]Green-Pedersen and Ebbesen (1997, Q.7 ll.125-6, p.36): *. . . numquam est formalis consequentia, si consequens significat plus quam antecedens.* Recall that Question 7 asks "whether in any possible scenario a false conclusion follows in a formal consecution from true premises", to which Drukken answers (ll.99-102, p.35): ". . . never in any scenario outside obligations and insolubles can a false conclusion follow from true premises such that the consecution was formal and the premises were true and the conclusion at the same time false" (*. . . ex materia obligationis et insolubilium numquam aliquo casu posito ex antecedente vero posset sequi consequens falsum, ita quod consequentia esset formalis et antecedens esset verum et consequens falsum simul*). We'll discuss Drukken's account of the exception in the case of insolubles in §2.1 below.

[24]*Ibid.* ll.103-5, p.35: *Sed si diceret aliquis, quod ista tota regula negaretur tibi et etiam Aristoteli, quomodo velles ipsam probare? Et puto quod impossibile est ipsam demonstrare, sed tamen sic potest deduci . . . .*

[25]*Loc.cit.* ll.105-24: *Omnis consequentia quae est bona est bona pro significato antecedentis et significato*

This account of formally valid inference goes back at least to Abelard.[26] A classic example of a formally valid consecution is 'Socrates is human, so Socrates is an animal', where the whole of what the conclusion signifies (Socrates and animality) is contained in what the premise signifies (Socrates and humanity).[27]

2 Insolubles

Containment may remind one of Bradwardine's notorious second postulate:

> "Every sentence signifies or means as-of-now or simply everything which follows from it as-of-now or simply."[28]

Bradwardine's solution to the insolubles relied on demonstrating that they mean more than at first appears. This comes out in his Second Thesis, that any sentence which signifies that it itself is false (or not true), also signifies that it is true. Consequently, any such sentence is false, since things cannot be wholly as it signifies.[29]

Bradwardine's solution to the insolubles marks a significant shift in the generally accepted response to the insolubles. Before Bradwardine presented his solution, the standard response to the insolubles was restrictionism (*restrictio*), that self-reference was impossible, at least in the context of privative terms like 'false' or ''not true'. In both restrictionism and cassationism (*cassatio*), which seems to have been more often criticized than adopted, it was taken for granted that insolubles signify at most what they appear to signify, and nothing more. For example, 'Socrates says a falsehood' signifies at most that Socrates says a falsehood and nothing more. To avoid contradiction, restrictionism insists that in such cases, the part (in this case, 'falsehood') cannot supposit for the whole of which it is part, so must either supposit for some other utterance of Socrates', or fail to supposit. In the latter case, which is the one most often entailed by the scenario (in which this is

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consequentis. Tunc vel significatum consequentis est idem cum significato antecedentis vel est aliquid aliud disparatum. Si aliquid aliud disparatum, tunc intellectus numquam est tali antecedente infert consequens non plus quam inferret: 'Homo currit. Ergo baculus stat in angulo', quia hoc vocatur consequentia, quando intellectus ex naturali iudicio suo ex antecedente infert consequens, et ex intellectu antecedentis intellegit significatum consequentis. Si dicatur quod significatum consequentis est idem quod significatum antecedentis, tunc vel est totaliter idem vel in aliquo et in aliquo non. Si in aliquo et in aliquo non, tunc dividitur significatum per imaginationem in excedens et in excessum. Tunc ratione illius excedentis consequens non infertur ex antecedente, quia illud excedens est totaliter disparatum a significato antecedentis. Ergo oportet quod totum significatum consequentis sit significatum antecedentis. Ergo si antecedens est verum, consequens est verum, quia ex quo significant idem penitus, ideo si una propositio est vera et etiam alia. Et hoc est quod dixerunt antiqui, quod in omni consequentia bona consequens includitur in antecedente, hoc est quod si est bona et formalis, totum significatum consequentis est significatum anteedentis, licet non e contra.

[26] See, e.g., Martin (1987). It is the fourth account of the truth of conditionals in Sextus Empiricus (1994, II 110-12), but although there are three surviving copies of a thirteenth-century Latin translation, Sextus's work only became widely known in the Latin West in the Renaissance: see, e.g., Wittwer (2016).

[27] See, e.g., Dutilh Novaes (2020, §2.2) and Bosman (2018).

[28] Thomas Bradwardine (2010, §6.3, p.96): *Suppositiones autem sunt sex ... Secunda est ista: quelibet propositio significat sive denotat ut nunc vel simpliciter omne quod sequitur ad istam ut nunc vel simpliciter.*

[29] Thomas Bradwardine (2010, §6.4, p.96): *Post hec omnia, ordine competenti secuntur due conclusiones ... Secunda est ista: si aliqua propositio significet se non esse veram vel se esse falsam, ipsa significat se esse veram et est falsa.*

Socrates' only utterance), his utterance has an empty term and so is false. Cassationism similarly rejects the possibility that the part supposits for the whole, but concludes that the utterance fails to signify at all, or at least, that no significant truth-apt sentence has been uttered.

2.1 Drukken on Insolubles

Some of the subsequent writers on insolubles resisted Bradwardine's proposal that insolubles, perhaps all sentences, have some further hidden meaning that only the shrewd logician can discern. Among them are Walter Segrave, defending restrictionism, John Dumbleton, defending a radical solution apparently combining aspects of both restrictionism and cassationism, and Roger Swyneshed, proposing that truth require that a sentence not falsify itself, which (as we noted in §1.1) he claimed insolubles do simply by entailing their own falsehood. The mainstream, led it seems by William Heytesbury, followed Bradwardine in accepting that insolubles have an additional signification besides what they appear on their face to signify, but for the most part rejecting Bradwardine's second postulate and the clever proof using it to prove his Second Conclusion.

From the late 1330s onwards, following publication of Heytesbury's *Rules for Solving Sophisms* in 1335, the standard solution popular in Oxford and the English Nation at Paris is found in such authors as Ralph Strode, John of Holland and John Hunter, and in many of the collective and often anonymous treatises in the *Logica Oxoniensis*. According to this solution, insolubles also signify that they themselves are true. For example, in an anonymous treatise ascribed by its editor to "pseudo-Heytesbury" on account of its similarity to Heytesbury's genuine work, we read (on the assumption that 'A falsehood exists' is the only proposition):

> "It must be said that ... 'A falsehood exists' signifies conjunctively, namely, that a falsehood exists and that that very proposition is true."[30]

This appears to be the solution favoured by Drukken. In a further argument in Question 7, he takes Socrates' utterance of 'Socrates says a falsehood' and nothing else to construct a valid consecution with true premise and false conclusion:

> "Sixthly, supposing that Socrates says only this true sentence: 'Socrates says a falsehood', then one argues like this: 'It is the case that Socrates says a falsehood, therefore Socrates says a falsehood'. The consecution is sound, because given the contradictory of the conclusion: 'Socrates does not say a falsehood', therefore it is not the case that Socrates says a falsehood, which is the opposite of the premise. Therefore the consecution was sound. Also, the premise is true in the scenario proposed, because in the scenario proposed it is the case that Socrates says a falsehood. But then I prove that the conclusion is false, namely, 'Socrates says a falsehood', because everything said by Socrates is false. But 'Socrates says a falsehood' was said by Socrates. Therefore, 'Socrates says

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[30]Pironet (2008, 292): "*Et eodem modo dicendum est ... ⟨quod⟩ 'falsum est' significat similiter copulative, videlicet quod falsum est et quod eadem propositio est vera.*"

a falsehood' is false. But this is the conclusion. Therefore the conclusion is false."[31]

The reasoning looks rather confused. But his solution seems to turn on accepting that Socrates' utterance signifies not only that it is false, but also that it is true. For Drukken then writes:

> "This conclusion can also be derived through the popular solution to insoluble sentences: and it is also clear by the rule which everyone grants on the subject of insolubles, which is that every sentence signifying itself to be true and itself to be false is simply false. But this sentence, 'Socrates says a falsehood', is of this sort, because in the scenario proposed it signifies that what is said by Socrates is a falsehood, and nothing else was said by Socrates except this sentence. Therefore it signifies that it itself is false. It also signifies that things are as is signified by it, and thence it signifies itself to be true. But because it is impossible that it is at the same time true and false, therefore it is impossible that things are wholly as is signified by it. And thence if it is not wholly the case as is signified by it, it is simply false. Therefore, Aristotle's rule is true, that every sentence signifying itself to be true and false is simply false. Therefore, this sentence, 'Socrates says a falsehood', is false. And this is the original conclusion. Therefore, the conclusion was false and the premise true, as was to be proved. Therefore etc."[32]

The idea that every sentence signifies or means (*significat vel denotat*) that things are as it signifies and so (implicitly) signifies its own truth is traced by Segrave to the function of the copula, as noted by Aristotle himself:

> "... every proposition means things to be in reality as it signifies. This is self-evident and is clear from the Philosopher and the Commentator in comment 14 on the fifth book of the Metaphysics and throughout the text of that comment: for the copula in the proposition signifies being true, as is elucidated there."[33]

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[31]Green-Pedersen and Ebbesen (1997, Q.7 ll.60-70, p.34): *Item sexto: Supponendo quod Socrates solum dicat veram talem propositionem: 'Socrates dicit falsum'. Tunc arguitur sic: 'Ita est quod Socrates dicit falsum. Ergo Socrates dicit falsum'. Consequentia est bona, quia detur contradictorium consequentis: 'Socrates non dicit falsum', ergo non est ita quod Socrates dicit falsum, quod est oppositum antecedentis. Ergo consequentia fuit bona. Antecedens etiam est verum hoc casu posito, quia hoc casu posito ita est quod Socrates dicit falsum. Sed tunc probo quod consequens sit falsum, scilicet: 'Socrates dicit falsum', quia omne dictum a Socrate est falsum. Sed hoc 'Socrates dicit falsum' est dictum a Socrate. Ergo hoc 'Socrates dicit falsum' est falsum. Sed hoc est consequens. Ergo consequens est falsum.*
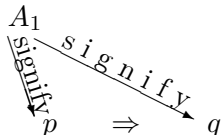
[32]*Loc.cit.* ll.71-86, p.34-35: *Posset etiam istud consequens deduci per communem positionem propositionis insolubilis; et hoc etiam patet per regulam, quam omnes concedunt de materia insolubili, et est quod omnis propositio significans se ipsam esse veram et se ipsam esse falsam est simpliciter falsa. Sed ista propositio: ' Socrates dicit falsum' est huiusmodi, quia isto casu posito significat quod aliquid dictum a Socrate est falsum, et nullum aliud est dictum a Socrate nisi istamet propositio. Ergo ipsamet significat, quod ipsa sit falsa. Ipsa etiam significat, quod sic est sicut per ipsam significatur, et tunc significat se esse veram. Sed quia impossibile est quod ipsa sit simul vera et falsa, ergo impossibile est, quod totaliter sic sit, sicut per ipsam significatur. Et tunc si non totaliter est ita sicut per ipsam significatur, ipsa est simpliciter falsa. Ergo regula Aristotelis est vera, quod omnis propositio significans se ipsam veram et falsam est simpliciter falsa. Ergo haec propositio: 'Socrates dicit falsum' est falsa. Et hoc est primum consequens. Ergo consequens fuit falsum et antecedens verum, ut probatum est. Ergo etc.*

[33]See Read (2023, p.50): *... quelibet propositio denotat ita esse ex parte rei sicut ⟨ipsa⟩ significat.*

2.2 Bradwardine's Second Postulate

Bradwardine's second postulate can be interpreted as a closure principle, that signification is closed under (simple or as-of-now) consequence, so that a sentence signifies anything that follows from anything it signifies. But Paul of Venice gives a different interpretation, one which connects interestingly with Drukken's fourth definition of validity.
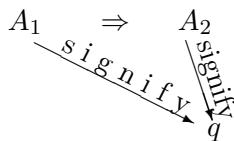
Elsewhere, I interpreted Bradwardine's postulate like this:[34]

that is, if A_1 signifies that p, and p entails q, then A_1 signifies that q. But Paul of Venice writes:

> "I say that any sentence signifies the significate of any sentence following from it formally ... This is how the common saying, 'Any sentence signifies whatever follows from it', should be understood."[35]

That suggests a different diagram:[36]

i.e., if A_2 signifies that q, and A_1 entails A_2, then A_1 signifies that q.[37] Putting them together, we have two "routes" from A_1 to q, a "northern route" (with Paul) and a "southern

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Hec est manifesta per se et patet per philosophum et commentatorem 5º metaphysice commento 14 et in littera illius commenti per totum: copula enim in propositione significat esse verum, ut ibi declaratur. The reference is to Aristotle's *Metaphysics* Δ 7, 1017a31: "'to be' and 'is' signify that a thing is true, and 'not to be' that it is not true but a falsehood, equally in the case of affirmation and of denial; as for instance that Socrates *is* artistic, that this is true, or that Socrates *is* not-pale, that it is true; and 'a diagonal *is not* commensurable' that it is a falsehood." (Aristotle, 1971, p.40)
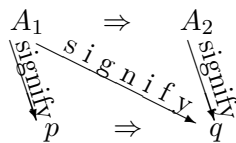
[34] For example, Thomas Bradwardine (2010, 'Introduction' §5) and Read (2015, p. 405).

[35] Paulus Venetus (1978, p.74): *Dico igitur quod quaelibet propositio significat cuiuslibet propositionis significatum ad ipsam formaliter sequentis ... Et sic debet intelligi illud commune dictum 'Quaelibet propositio significat quidquid sequitur ad eam' ....*

[36] Note that there is an abuse of notation here. In $p \Rightarrow q$, '$\Rightarrow$' is an operator or connective, connecting what is signified by sentences (propositions), whereas in $A_1 \Rightarrow A_2$, '$\Rightarrow$' is a relation between those sentences. On the harmless need to move back and forth between these two grammatical forms, see Anderson and Belnap (1975, §A5, pp. 80-2).

[37] A point of clarification made by Ralph Strode in his *Insolubilia* (ms Erfurt Amploniana Q255, fol.9ra) suggests that he takes Bradwardine's second postulate in this way: "All the [earlier] postulates are true except the second, which is false taken literally, namely: 'Every sentence signifies or means as-of-now or simply everything which follows from it as-of-now or simply', because if so, it would follow that 'Socrates is running' would signify 'A substance exists', which is self-evidently false. Nevertheless, if it is understood [as saying] that every sentence signifies or means as-of-now or simply every significate of a sentence which follows from it as-of-now or simply', then it can tolerably be sustained: e.g., the sentence 'A man is running' signifies that an animal is running and that an animal exists and that a substance is running and that a substance exists, but it does not signify the sentences 'An animal is running', 'An animal exists', 'A substance is running', 'A substance exists' and so on. And this is perhaps how the author understood it."

route' (the closure condition):

$$A_1 \quad \Rightarrow \quad A_2$$

$$p \quad \Rightarrow \quad q$$

2.3   Dumbleton's Counterexamples

John Dumbleton claimed that Bradwardine's second postulate

> "should not be maintained wholly and universally without qualification, since
> there are some necessary consecutions whose conclusions do not signify as their
> premises do. And there are other formal consecutions whose conclusions do not
> mean the same as their premises."[38]

Dumbleton gave two counterexamples to Bradwardine's second postulate, first:

> Some man is an ass, therefore some man is a goat

and secondly, more generally, that from every affirmative proposition it follows that God
exists.[39] He claims that the first is valid, since it is impossible that any man is an ass—
nonetheless, there is no necessary relation between the premise and the conclusion. The
second is also valid, he says, presumably because the conclusion is necessary—but again,
"not every affirmative proposition signifies that God exists", contrary to Bradwardine's
postulate. The advocate of Containment as a criterion of validity would obviously deny

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(*Omnes suppositiones sunt vere preter secundam, que falsa est de virtute sermonis, ista scilicet 'quelibet
propositio significat vel denotat ut nunc vel simpliciter omne quod sequitur ad ipsam ut nunc vel simpliciter',
quia sic sequitur quod ista propositio 'Sortes currit' significaret istam 'substantia est', quod de se patet esse
falsum. Verumtamen si intelligatur quod omnis propositio ut nunc vel simpliciter significat vel denotat ut
nunc vel simpliciter omne significatum propositionis ad ipsam sequens ut nunc vel simpliciter, tunc potest
tollerabiliter sustineri: sicud ista propositio 'homo currit' significat quod animal currit et quod animal est
et quod substantia currit et quod substantia est, et sic vlterius, non tamen significat istas propositiones
'animal currit', 'animal est', 'substantia currit', 'substantia est', et sic ultra. Et sic forte intellexit autor*).

[38] John Dumbleton (202x, §A3.0): ... *est notandum quod cum in quibusdam tractatibus dicitur omnem
propositionem significare quicquid sequitur ad eam ut nunc vel simpliciter, hoc non in toto et universaliter
simpliciter est sustinendum cum quedam sunt consequentie necessarie quarum consequentia non significant
ut antecedentia. Et alie sunt formales per quarum consequentia non denotatur idem quod per antecedentia.*

[39] John Dumbleton (202x, §§A3.1-3.2.1): "For the first case, take this consecution: 'Some man is an ass,
therefore some man is a goat', which is said to be necessary since it cannot be that some man is an ass
unless he is a goat, so the consecution is necessary, although there is no necessary relation between the
premise and the conclusion ... Moreover, from every affirmative sentence it follows that God exists and
not every affirmative sentence signifies that God exists, as is clear from this principle: every affirmative
sentence signifies ⟨in some way⟩ solely for that thing which is signified by its subject" (*Pro primo: capiatur
talis consequentia: 'Homo est asinus, ergo homo est capra,' que necessaria dicitur cum non potest esse
quod homo sit asinus nisi sit capra, quare necessaria consequentia est, licet non sit necessaria habitudo
inter antecedens et consequens ... Item ex omni propositione affirmativa sequitur deum esse et non omnis
propositio affirmativa significat deum esse ut patet per hoc principium: omnis propositio affirmativa pro re
tali solum significat que significatur per eius subiectum*).

that these consecutions are valid, or at least, formally valid. Many authors include them as so-called "materially valid" consecutions.[40] Others, including Drukken, dismiss them as invalid, precisely because they contravene the containment condition. Drukken writes:

> "But I consider that none of them is a sound consecution, because in every sound consecution what the premises and the conclusion signify is the same, so that whatever is signified by the conclusion should be signified by the premises, and then the understanding by a natural judgment infers the conclusion from the premise."[41]

Consequently, he believes that every sound consecution is formal.[42] One might object, however, that Drukken has misidentified the necessity which the premises invoke. They don't necessitate that one infer, assert or even consider, the conclusion. What they necessitate is that one not assert the opposite of the conclusion (as noted in the third definition, the incompatibility criterion). Given the premises, the understanding recognises that they contain the conclusion, and as such permit its assertion, but more importantly, they exclude the assertion of its contradictory.

2.4   Bradwardine's Second Thesis

If we interpret Bradwardine's Second Postulate as Paul does, then it is entailed by Containment. For we can formalize Containment as follows:

$$A_1 \Rightarrow A_2 \text{ iff } \forall p(\text{if } \mathbf{Sig}(A_2, p) \text{ then } \mathbf{Sig}(A_1, p))$$

which, by permuting the antecedents, clearly entails:

$$\forall p(\text{if } \mathbf{Sig}(A_2, p) \text{ and } A_1 \Rightarrow A_2 \text{ then } \mathbf{Sig}(A_1, p))$$

which is a formalization of Paul's interpretation of the Second Postulate. We then need to take Bradwardine's argument for his Second Thesis (that any sentence that signifies

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[40] Just as according to the English account, formal validity is broader than on the Parisian account, so material validity is narrower. Drukken writes (Green-Pedersen and Ebbesen, 1997, Q.9 ll.164-7, p.49): "But some propose this distinction, that some simple consequence is material, some formal. And material [consequence] is what holds solely in virtue of the terms, and holds by these two rules: From the impossible anything follows; and: The necessary follows from anything" (*Sed alii ponunt talem distinctionem, quod consequentia simplex alia est materialis, alia formalis. Et materialis est quae solum tenet ratione terminorum, et tenet per istas duas regulas: Ad impossibile sequitur quodlibet; et: Necessarium sequitur ad quodlibet*). See e.g., Ockham (1974, III 1, p.589), corrected against Schupp (1993): "Material consequence is when it holds precisely in virtue of the terms and in virtue of some extrinsic middle precisely respecting the general conditions of propositions. Of this sort are 'If a man runs, God exists', 'A man is a donkey, therefore God does not exist', and so on" (*Consequentia materialis est quando tenet praecise ratione terminorum et ratione alicuius medii extrinseci respicientis praecise generales condiciones propositionum; cuiusmodi sunt tales 'si homo currit, Deus est'; 'homo est asinus, igitur Deus non est' et huiusmodi*).

[41] Green-Pedersen and Ebbesen (1997, Q.9 ll.170-75, p.50): *Sed puto quod nulla illarum est bona consequentia, quia in omni bona consequentia significatum antecedentis et consequentis sunt idem, ita quod quicquid significatur per consequens debet significari per antecedens, et tunc intellectus naturali iudicio ex antecedente infert consequens.*

[42] Green-Pedersen and Ebbesen (1997, Q.14 ll.118-9, p.81): *Sed breviter omnis consequentia bona formalis est, ut est dictum prius.* See also (1997, Q.4 ll.80-4, p.21).

itself to be false also signifies itself to be true) and check that it is compatible with Paul's interpretation.

The proof has two parts: first, suppose that some sentence, A, signifies only that it itself is false and nothing else, and suppose A is false. Then things are not as it signifies, so it's true, that is, $FA \Rightarrow TA$. But 'FA' is A, so $A \Rightarrow TA$. Since TA signifies that A is true, A must also signify that it is true (by Paul's version of Bradwardine's postulate). So A does not signify only that it is false, but also that it is true.[43]

That takes us to the second part of Bradwardine's proof, which is the nub of his argument: So A signifies more than that A is false, but also, say, that b is c. Again, suppose A is false. Then things are not as A signifies, so either it's true or b is not c, that is, $FA \Rightarrow (TA \vee b \text{ is not } c)$. Now A is '$FA \wedge b$ is c', and given that $FA \Rightarrow (TA \vee b$ is not $c)$, $(FA \wedge b \text{ is } c) \Rightarrow ((TA \vee b \text{ is not } c) \wedge b \text{ is } c)$, and so A (that is, $FA \wedge b$ is $c) \Rightarrow TA$. Hence, as before, A must also signify that it is true (by Paul's version of Bradwardine's postulate).[44]

Thus any sentence that signifies that it is itself false, also signifies that it is true, and so, being implicitly contradictory, must be false. Not everything such an insoluble sentence signifies can obtain, and so every insoluble is false.

2.5 Revenge

Solutions to the insolubles such as Bradwardine's sail close to the wind, however, and one might suspect that paradox has returned in that argument of Bradwardine's, when he claims that $A \Rightarrow TA$ (from which he infers $\mathbf{Sig}(A, TA)$). For A is just FA, and

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[43]Thomas Bradwardine (2010, §6.6.1, pp.102-3): "The first part of the second thesis is proved fully: let A be a sentence signifying itself not to be true, whether signifying anything else or not. If not, then it follows from the assumption that A is not true, by the first definition, that it is not wholly as is signified by A, and by A is signified only that A is not true (we supposed), so by the second part of the first thesis it follows that it is not the case that A is not true, that is, that A is true, whence by the second postulate A signifies itself to be true" (*Secunda conclusio quantum ad primam sui partem totaliter demonstratur. Sit a propositio significans se non esse veram: vel ergo significat aliquod aliud vel non. Si non, et sequitur: a non est verum, ergo non est ita totaliter sicut significatur per a per primam diffinitionem, et per a tantum significatur a non esse verum, ut ponitur, ergo per secundam partem prime conclusionis, non est ita quod a non est verum, ergo est ita quod a est verum, ergo per secundam suppositionem a significat se esse verum*).

[44]*Loc.cit.*: "If ⟨on the other hand⟩ A signifies more than that it itself is not true, for example, that b is c, then supposing that A is not true, it follows that it is not wholly as A signifies, and A signifies only that A is not true and b is c, whence it is not the case that A is not true and that b is c, and so, by the fourth postulate, A is true or b is not c, and so, by the second postulate, A signifies that A is true or b is not c, and since it signifies the opposite of the second part of this disjunction (we supposed), namely, that b is c, we have by the fifth postulate, from this disjunction with the opposite of its second part that the first part follows, namely, that A is true. Hence by the second postulate, A signifies A to be true. We should argue in such a fashion whenever A signifies itself not to be true and many other things" (*Si a significaret aliud quam se non esse verum, sit illud: b est c, tunc sequitur: a non est verum, ergo non est ita totaliter sicut significatur per a, et per a tantum significatur a non esse verum et b est c, ergo non est ita quod a non est verum et quod b est c, ergo per quartam suppositionem, a est verum vel nullum b est c, ergo per secundam suppositionem, a significat quod a est verum vel quod nullum b est c, et significat oppositum secunde partis huius disiunctive, ut ponitur, scilicet quod b est c et per quintam suppositionem, ex ista disiunctiva cum opposito secunde partis sequitur prima, scilicet quod a est verum. Ergo per secundam suppositionem a significat a esse verum. Consimiliter omnino est arguendum, si ponatur a significare se non esse verum et multa alia*).

Bradwardine claims that A is false (that is, FA), and from $FA \Rightarrow TA$ and FA it follows immediately that TA, that is, that A is not only false, but also true, and paradox has returned.

This is a general problem for all those solutions, including Bradwardine's, the modified Heytesbury, Albert of Saxony's, Marsilius of Inghen's and even John Buridan's,[45] which propose that each insoluble, or perhaps every sentence, additionally signifies or, in Buridan's case, implies its own truth. For any sentence which signifies its own truth must surely entail it too. Somehow, a revenge paradox must be avoided, whereby accepting any form of what has come to be known as Capture, that is, $\alpha \Rightarrow T\alpha$,[46] contradicts the verdict that every self-falsifying sentence is false.

This problem does not affect the first leg of Bradwardine's proof of his Second Thesis, despite appearances. For the premise of the application of the Second Postulate, of the form $A \Rightarrow TA$ (that is, $FA \Rightarrow TA$, since A is FA), depends on the, as it turns out, false assumption that A signifies only that it itself is false. However, that is not so in the second leg of the argument, where it appears to be asserted categorically that $A \Rightarrow TA$. Bradwardine responds as follows, applied to the specific case where Socrates utters 'Socrates says a falsehood':

> "But if it is true that Socrates utters a falsehood, and Socrates says that, then Socrates utters a truth.
> The reply is that the minor premise is false, because he does not utter the proposition presented by you, and conceded by me, but another ⟨sentence⟩ like it ..."[47]

As we noted in §2.4, under Paul's interpretation, what entails TA is '$FA \wedge b$ is c', where clearly 'b is c' (perhaps better written predicatively as 'B is C', or in fact 'A is T' or 'TA') includes the further signification that Bradwardine's reasoning has revealed. The sentence Socrates utters is self-referential and so signifies differently from Bradwardine's own equiform utterance, which is not self-referential and refers not to itself but to Socrates' utterance. Having a different signification, their truth-conditions are different, the one false, the other true.

Drukken makes the same distinction in his response to the sixth puzzle in Question 7:

> "Turning to the proof, you say: "This is false: 'Socrates says a falsehood'." I grant that this sentence, which Socrates says, is simply false, and with this, I grant that that sentence is not the conclusion from that initial premise. And for this reason, if then there was no one except Socrates in the scenario proposed, then no mental consecution would be sound because there would be only one sentence alone in Socrates' mind. And for this reason I readily grant that this consecution which I make, that it is sound. But then both the premise and the conclusion supposit for the sentence which Socrates says, and each is

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[45] On these solutions, see Spade and Read (2021, §§3.1, 3.5, 3.8, 3.9, 4.1).

[46] See, e.g., Beall (2007, pp.1-3).

[47] Thomas Bradwardine (2010, §7.1.1-ad 7.1.1): *Contra: tunc hec est vera: Sortes dicit falsum, et Sortes dicit istam, ergo Sortes dicit verum. Dicendum quod minor est falsa, quia non dicit istam propositionem propositam a te et concessam a me, sed aliam sibi similem ....*

true, although Socrates' sentence is simply false. And for this reason, when one argues: "everything said by Socrates is false," I grant it, and "Socrates' sentence" etc., I grant what was concluded. Then further: "This sentence is false, and this was the conclusion." I deny the minor premise, because it was not the conclusion, but another sentence which is mine, and the one I have is similar to the one which is Socrates'. But mine is true, and Socrates' is false. And thus in no such scenario does it follow. Whence etc."[48]

So too in the case of the version of Bradwardine's proof of his Second Thesis in §2.4: Bradwardine makes the assumption that A is false. That assumption is not A itself, but a meta-linguistic statement about A. He is not assuming that his assumption is false, but that Socrates' utterance is false. Bradwardine then shows that A, that is '$FA \wedge b$ is c', entails that A is true, and so, by the Second Postulate, it follows A signifies that A is true, and so, signifying contradictory things, A must be false.

2.6 Signification as-of-now

There is one final complication, however. Recall Bradwardine's statement of his second postulate:

> "Every sentence signifies or means as-of-now or simply everything which follows from it as-of-now or simply."

Containment, at least as stated by Drukken, applies only to simple consequence (*consequentia simplex* or *simpliciter*), since Drukken rejects the notion of consequence "as-of-now" (*ut nunc*). The reason he rejects it is the same as his reason for rejecting material consequence, namely, that consequence *ut nunc* is enthymematic, and so our understanding of what is signified by the premises does not suffice for the natural judgment needed to infer the conclusion. Take the claim that:

> Socrates is not running now. Therefore, a man is not running,

is valid as-of-now, supposing that Socrates is now a man, since "for some present moment, the premise cannot be true unless the conclusion is true, so the consecution is sound as-of-now."[49] Drukken responds:

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[48]Green-Pedersen and Ebbesen (1997, Q.7 ll.237-51, p.40): *Ad probationem: Tu dicis: "Hoc est falsum: 'Socrates dicit falsum'." Concedo quod haec propositio, quam Socrates dicit est simpliciter falsa, et cum hoc concedo, quod illa propositio non est consequens ad primum antecedens. Et ideo si tunc nullus esset nisi Socrates hoc casu posito, tunc nulla consequentia mentalis esset bona, quia non esset nisi una sola propositio in mente Socratis. Et ideo bene concedo, quod ista consequentia, quam facerem, quod ipsa esset bona. Sed tunc tam antecedens quam consequens supponunt pro illa propositione, quam dicit Socrates, et utrumque est verum, licet propositio Socratis est simpliciter falsa. Et ideo quando arguitur: "Omne dictum a Socrate est falsum," concedo, et "Propositio Socratis est" etc., concedo conclusionem. Tunc ultra: "Haec propositio est falsa, et hoc fuit consequens." Nego minorum, quia illa non fuit consequens, sed una alia quam habeo, et illa quam habeo est similis illi, quam Socrates habet. Tamen mea est vera, et illa Socratis est falsa. Et sic in nullo tali casu sequitur. Quare etc.* (See also Q.9 ll.321-36, p.55.)

[49]Green-Pedersen and Ebbesen (1997, Q.9 ll.131-5, p.48): *'Nunc Socrates non currit. Ergo homo non currit', scilicet supposito quod Socrates nun sit homo ... quia pro aliquo praesenti antecedens non potest esse verum, nisi consequens sit verum. Ideo ut nunc talis consequentia est bona.*

> "if your explanation were valid, namely, that the premises cannot be true without the conclusion, then for the same reason you would have to concede that any true sentence would imply any other truth, and so 'You are sitting, therefore your friend is sitting' would be a sound consecution, and so on for others."[50]

Consequently, he says, "All consequence is simple,"[51] that is, every valid consecution follows simply and only simple consecutions are valid.

But including signification as-of-now in his second postulate is crucial to Bradwardine's solution to the insolubles, in order to deal with contingent paradox. For example, if Socrates says only 'Socrates says a falsehood', it follows only as-of-now that it is itself false, in virtue of the contingent fact that it is the only sentence he utters. Bradwardine infers from its following as-of-now that it is true that it also signifies as-of-now that it is true. Robert Eland similarly connects the two notions when he characterises convertibility as-of-now:

> "Sentences are convertible as-of-now when one signifies just as the other as-of-now."[52]

This suggests a characterisation of as-of-now consequence in terms of containment of what the conclusion signifies as-of-now in what the premises signify as-of-now. Accordingly, following simply and following as-of-now would, between them, support the full force of Bradwardine's second postulate for application to every insoluble, contingent or not.

But recall that consequence as-of-now is enthymematic. For most authors that is unproblematic: consequence is an objective matter for those who charactise it in terms of truth-preservation, or verification-preservation, or even incompatibility. But for someone like Drukken who defines consequence in terms of containment of the understanding of the conclusion in that of the premises, the idea that hidden and unstated additional facts might validly connect premises and conclusion is anathema. Even more so is it with signification as-of-now. He agrees with what Dummett once said about meaning and understanding, that meaning should be transparent: if two expressions which one understands mean the same, then one must know that they mean the same.[53] That ability extends for Drukken to the ability to articulate the soundness of a consecution in a "natural judgment [that] infers the conclusion from the premises."

This is a thoroughly internalist conception of signification and consequence. It stands in contrast to the implicit semantic externalism of Bradwardine's conception of those notions, which accepts, indeed proclaims, that what follows from our assertions, and indeed, even what they signify, may extend well beyond our understanding and awareness. Socrates

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[50]Green-Pedersen and Ebbesen (1997, Q.9 ll.146-9, p.49): *... quia si tua causa valeret, scilicet quod antecedens non potest esse verum sine consequente, tunc eadem ratione haberes concedere, quod quaelibet propositio vera inferret aliam veram, et sic esset bona consequentia: 'Tu sedes. Ergo tuus socius sedet', et sic de aliis.*

[51]Green-Pedersen and Ebbesen (1997, *loc.cit.*): *Omnis consequentia est simplex.*

[52]Spade (1976, §5, p.58): *Propositiones convertuntur ut nunc quando una significat sicut alia ut nunc.* On the identity of Eland, see Read and Thakkar (2016).

[53]Dummett (1978a, p.131) argued that Frege distinguished sense (Sinn) from reference (Bedeutung) since the former is transparent whereas the latter is not: "It is an undeniable feature of the notion of meaning [i.e. sense, Sinn]—obscure as that notion is—that meaning is *transparent* in the sense that, if someone attaches a meaning to each of two words, he must know whether these meanings are the same."

innocently said that what he said was false. He thought he knew what his utterance signified: he knew its usual signification. But it took theorists such as Aristotle,[54] and Bradwardine and his successors to claim that, to Socrates', and the man on the omnibus's astonishment, it also signified its own truth. They may or may not have been right; but a presupposition of their approach to the insolubles was that they could claim, and hopefully, back with reasons, that part of what ordinary speakers signified by their utterances was beyond their ken, at least until it was pointed out and explained to them. "Meanings just ain't in the head", as Putnam quipped.[55]

Summary

Niels Drukken of Denmark discussed five different ways of defining validity, or consequence. Buridan and Swyneshed, in their different ways, cast doubt on truth-preservation as a defining feature of validity. Buridan proposed preservation of signifying-as-things-are (*aka* verification) as the basis of validity. But John Mair presented a further puzzle that seemed to show that verification-preservation would not suffice. Many authors, including Girard Odo and Paul of Venice, suggested that the incompatibility of the opposite of the conclusion with the premises was the correct mark of validity. But that seems circular, since incompatibility itself is defined in terms of validity. Finally, the containment of the conclusion in the premises, that is, that the premises should signify whatever is signified by the conclusion, seems to capture validity.

Interestingly, Bradwardine's famous Second Postulate, as interpreted by Paul of Venice, follows from the characterization of validity by the Containment condition. Although John Dumbleton argued against this postulate, his objections assume the validity of the strict implication paradoxes, which many, including Drukken, reject; and Bradwardine's proof of his Second Thesis, solving the logical paradoxes (that is, the insolubles), is consistent with Paul's interpretation of the postulate. Nonetheless, such solutions as Bradwardine's must avoid a commitment to Capture, whereby any sentence entails its own truth, on pain of contradiction. Moreover, Bradwardine's inclusion of as-of-now consequence in his postulate shows how far his externalist conception of signification lies from Drukken's and perhaps others' reason for endorsing the containment criterion.

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[54]When surveying the various meanings of 'is', Aristotle (*Metaphysics* Δ 7, 1017a31-35) observes: "Again, 'to be' and 'is' signify that a thing is true, and 'not to be' that it is not true but a falsehood, equally in the case of affirmation and of denial; as for instance that Socrates *is* artistic, that this is true, or that Socrates is not-pale, that it is true; and 'a diagonal *is not* commensurable' that it is a falsehood."

[55]See Putnam (1975, p.227).

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