

# Paradox, Closure and Indirect Speech Reports

Stephen Read

**Abstract.** Bradwardine's solution to the the logical paradoxes depends on the idea that every sentence signifies many things, and its truth depends on things' being wholly as it signifies. This idea is underpinned by his claim that a sentence signifies everything that follows from what it signifies. But the idea that signification is closed under entailment appears too strong, just as logical omniscience is unacceptable in the logic of knowledge. What is needed is a more restricted closure principle. A clue can be found in speech act pluralism, whereby indirect speech reports are closed under intersubstitutivity of co-referential terms. The conclusion is that solving the semantic paradoxes does not require revision of logic, thus saving logic from paradox.

**Mathematics Subject Classification (2010).** 03A05,01A35.

**Keywords.** Paradox, Signification, Multiple-meaning, Bradwardine, Omniscience.

The solution put forward by the medieval logician, Thomas Bradwardine, to the insolubles (the logical paradoxes) depends on the thesis of multiplicity of meaning, that every sentence signifies many things, and any sentence's truth depends on things' being wholly as it signifies. This multiplicity of meaning is underpinned by Bradwardine's claim that signification is closed under consequence, that is, that a sentence signifies everything that follows from anything it signifies.

The idea of multiplicity of meaning has been gaining attention in philosophy of language and semantics in the last few years, in, e.g. Cappelen and Lepore's "speech act pluralism" and Cian Dorr's "propositional profusion". But the idea that signification, or meaning, is closed under logical consequence appears too strong and open-ended, just as logical omniscience is implausible as a principle of formal epistemology. What is desired in both cases is a more restricted claim of closure under some narrower notion of consequence. The clue can be found in speech act pluralism, based on the idea of

indirect speech reports. Such reports allow considerable latitude in describing what was said. For example, if Jack said that Justine bought a picture, and Justine was in fact French, then it is not wrong to report Jack as having said that someone bought a picture, and even that a French woman bought the picture, despite the fact that Jack might not have known that Justine was French.<sup>1</sup> But it would be wrong to report Jack as having said that everyone who came into the shop was French, even if Justine was the only customer that morning, even though it does follow from what Jack said. He certainly didn't say everything that follows from what he said, but nonetheless, indirect speech reports are closed under some restricted form of consequence.

## 1. Bradwardine's "Multiple-meanings"-solution

I argued elsewhere that we can use the idea behind speech act pluralism to give modern clothing to Bradwardine's medieval solution to the logical paradoxes. Bradwardine's novel insight was that each of the paradoxical statements implicitly signifies, says or means more than at first appears. He claims that in general, any sentence signifies everything that follows from anything it signifies: signification, or meaning, is closed under consequence. This is the second of six postulates.<sup>2</sup> But he does not say much in defence of this claim, other than what is implicit in its utility in diagnosing the fallacy in the logical paradoxes and providing a solution to them. He uses the closure principle to show that any sentence saying of itself that it is false also signifies that it itself is true, and so is implicitly contradictory. Consequently, not everything it signifies can obtain, and so it is false since things are not wholly as it says they are.

The argument is this:<sup>3</sup> take any sentence that means that it itself is false (e.g. 'This sentence is false')—call it  $A$ . Suppose that is all  $A$  means, and suppose  $A$  is false. Then things are not as it says or means they are, so  $A$  is not false but true. Thus, supposing  $A$  is false, it follows that it is true. But it says it's false, so by the closure principle, it says it's true. Consequently, it doesn't just mean that it is false, it also means that it's true. In particular, no sentence can mean only that it itself is false.

So suppose  $A$  means more than just that  $A$  is false—suppose it means that  $A$  is false and also that  $\phi$ , say. Again, supposing  $A$  is false, it follows that things are not wholly as it says they are, so either not- $\phi$  or  $A$  is not false but true. But if not- $\phi$  or  $A$  is true, and  $\phi$ , then  $A$  is true. So if  $A$  is false and  $\phi$ ,  $A$  is true. But  $A$  means that  $A$  is false and  $\phi$ , so again by the closure principle,  $A$  means that  $A$  is true.

Consequently,  $A$  means both that it is false and that it is true. But it's impossible that  $A$  is both true and false, so things can't be wholly as  $A$  says they are, so  $A$  is false. But although  $A$  means that  $A$  is false, and it is false,

<sup>1</sup>The example comes from Cappelen and Lepore (2005, p. 202).

<sup>2</sup>See Bradwardine (2010, ¶6.3, p. 97).

<sup>3</sup>See Bradwardine (2010, ¶¶6.6.1-5, p. 103).

we cannot infer that  $A$  is true. For we have shown that  $A$  means more than just that  $A$  is false, and to be true, things must be wholly as it says they are, and that is impossible. So  $A$  is simply false and not true. This is the “multiple-meanings” solution to the paradoxes.

Bradwardine’s result is a remarkable one, giving an unusual diagnosis of the underlying cause of the problem arising from the semantic paradoxes. Consider, for example, Robert L. Martin’s presentation of the Liar paradox and possible solutions to it in the ‘Introduction’ to his classic collection of 1984:<sup>4</sup>

“The problem is this: there are good reasons to accept as true the following two claims, *and* their incompatibility! The first claim is:

(S) There is a sentence that says of itself only that it is not true

...

It is hard to reject (S) in the face of a sentence such as ‘This very sentence is not true’ ...

The second claim is:

(T) Any sentence is true if, and only if, what it says is the case.”

Martin proceeds to consider four diagnoses, only the last of which challenges (S), and then only on the ground that it depends on a mistaken belief that sentences can describe their own truth or falsehood.

(S) had already been challenged by Arthur Prior in 1958, referring to Epimenides the Cretan:<sup>5</sup>

“We thus reach the peculiar conclusion that if any Cretan does assert that nothing asserted by a Cretan is true, then this cannot possibly be the only assertion made by a Cretan—there must also be, besides this false Cretan assertion, some true one.”

What Prior does not consider, however, is that this additional Cretan assertion might not require a further utterance, but might have already been made by Epimenides when he uttered the sentence ‘All Cretans are liars’. This, however, was the conclusion that Bradwardine was led to. Bradwardine, or at least the author of an additional chapter of his *Insolubilia* found in two of the manuscripts, considers an objection:<sup>6</sup>

“Suppose that  $A$  [‘Socrates utters a falsehood’, assumed to be Socrates’ only utterance] signifies only that Socrates utters a falsehood, then the general argument breaks down.”

Not so, he replies:

“It must be said that  $A$  signifies other than that Socrates utters a falsehood,”

for other things follow from  $A$ , e.g., that Socrates exists and that something is false, so  $A$  must signify them too, by the closure principle. Indeed, Bradwardine’s argument shows that, since  $A$  signifies its own falsehood (given that  $A$

<sup>4</sup>Martin (1984, p. 1).

<sup>5</sup>Prior (1958, p. 261). Cf. *Titus* 1:12.

<sup>6</sup>Bradwardine (2010, A.4, p. 183).

was Socrates' only utterance), *A* also signifies its own truth. *A* signifies many things. Meaning is multiple, and Martin's claim (**S**) is false. No sentence can say of itself only that it is not true.

## 2. Propositional Profusion

John Hawthorne and Cian Dorr claim that

“when one sincerely utters a declarative sentence, one typically asserts a vast number of different propositions (all of which one believes).”<sup>7</sup>

This recognition that what is said is manifold has begun to be explored in recent years. Dorr (2011) dubs it “propositional profusion”.<sup>8</sup> One way in which the idea is articulated couples semantic minimalism with semantic pluralism: that there is a minimal semantic content to any utterance, which can then be exploited to make any of a range of different assertions. This was the idea behind Cappelen and Lepore's speech act pluralism, of which the “central observation” was that “No one thing is said (or asserted, or claimed, or . . .) by any utterance: rather, indefinitely many propositions are said, asserted, claimed, stated, etc.”<sup>9</sup> Chalmers too suggests that

“two-dimensionalism is naturally combined with a semantic pluralism, according to which expressions and utterances can be associated with many different semantic (or quasi-semantic) values, by many different semantic (or quasi-semantic) relations.”<sup>10</sup>

However, Chalmers' pluralism is not Dorr's profusion, as is indicated by his talk of “different semantic relations”, and by his description (Chalmers, 2010, p. 383) of his “content pluralism” as one where “different sorts of content and the corresponding content relations may have a role to play for different explanatory purposes.” The reason is that Hawthorne and Dorr's “profusion” is not pluralist, but absolutist: one asserts and believes all the similar propositions involved.

What is the principle behind the generation of the different assertions that can be made with any utterance? Hawthorne and Dorr ask us to

“consider, for example, a view according to which each sentence is associated with a unique semantic content relative to a given context, but that an utterance of that sentence counts as an assertion not just of the proposition that is the semantic content, but of every proposition it entails. Implausible though it is, some theorists do talk as if assertion (and belief) were closed under entailment in this way.”<sup>11</sup>

<sup>7</sup>Hawthorne and Dorr (2011), p. 40.

<sup>8</sup>See Dorr (2011).

<sup>9</sup>Cappelen and Lepore (2005), p. 199.

<sup>10</sup>Chalmers (2010), Appendix: Two-Dimensional Semantics, p. 556.

<sup>11</sup>Hawthorne and Dorr (2011), p. 41.

Implausible it may be, yet Scott Soames has advocated a view like this in his “cluster theory” of meaning, whereby

“the meaning (or semantic content) of a ... sentence ... counts as asserted only if [it] is an obvious, relevant, necessary and a priori consequence of enriched propositions asserted in uttering [that sentence].”<sup>12</sup>

For Soames, as for Cappelen and Lepore, it is implicature rather than entailment that connects the primary, or minimal, semantic content with the indefinite cluster of enriched assertions that can be made:

“Someone who assertively utters a sentence often succeeds in asserting more than the proposition semantically expressed by the sentence in the context ... I should say a word about the difference between what is asserted by an utterance and what is merely conveyed, or implied. In both cases the speaker commits himself to a proposition. However, the nature of the commitment is different.”<sup>13</sup>

As examples, Soames describes someone remarking that “C has lied to us before”, thus implying that C cannot be trusted to tell the truth. But that was not what he actually said. In another example, Soames rehearses one of Grice’s examples of conversational implicature, when the speaker remarks that “there’s a gas station around the next corner”, thereby implying that he believes it’s open and selling gas.<sup>14</sup> But, says Soames, “this ... is not something that [he] says or asserts.” He goes on to contrast the “semantic content of the sentence uttered in the context” with what the speaker says, implies, hopes and entails, and the information the audience acquires. The semantic content is minimal; the semantic import is much broader.

Soames, and Cappelen and Lepore, distinguish the semantic content from the pragmatic implicatures that content provokes. Bradwardine, like Hawthorne and Dorr, consider this cloud, or chain, to be one of entailment and so part of the semantic content. It took the genius of Bradwardine to realise that Socrates (and Epimenides) had also said that they were telling the truth as well as lying, which, it seems, Epimenides had not realised (nor had St Paul realised that Epimenides had called himself a liar either, perhaps). But the chain continues, nonetheless. Yet how far does it extend? Does a sentence really signify everything that follows from what it signifies?

### 3. Logical Omniscience

Jaakko Hintikka’s account of knowledge and belief notoriously attributed logical omniscience to every knower, by supposing that knowledge is closed

<sup>12</sup>Soames (2008, p. 257).

<sup>13</sup>Soames (2002, ch.3: ‘The Meaning of Names’, pp. 79, 84).

<sup>14</sup>See Grice (1975, p. 51).

under logical consequence. Hintikka analysed knowledge in terms of epistemic alternatives:

‘*a* knows that *p*’ is true at *w* iff *p* is true at all epistemic alternative to *w*,

“i.e., in all the epistemically possible worlds which are compatible with everything *a* knows in *w*.”<sup>15</sup> Now suppose that *p* entails *q*, and that *a* knows that *p*. Let *w*’ be an epistemic alternative for *a*. Then *p* is true in *w*’, and since *p* entails *q*, *q* is true in every world in which *p* is true, so *q* is true in *w*’ too. Hence *a* knows that *q*, since *w*’ is an epistemic alternative for *a*. Thus knowledge is closed under entailment, and *a* is logically omniscient.

In his original account,<sup>16</sup> Hintikka accepted this consequence, effectively saying that he was interested only in ideal, that is, deductively omniscient knowers. He subsequently realised that this response was inadequate. As Stalnaker (1991, p. 428) observes, the agents we are interested in use reasoning, and “reasoning (at least deductive reasoning) is an activity that deductively omniscient agents have no use for.” Logic as a means of inferring one thing from another would be meaningless for such a knower. So Hintikka proposed to extend the model theory with what he called “impossible possible worlds”. Not all epistemic alternatives are logically possible—for the agent *a* may not be an ideal knower. Hintikka used Rantala’s variant urn-models to represent such an agent’s knowledge and belief, an agent who can only analyse quantifiers to a certain depth. Such models are in fact logically inconsistent, but their inconsistency only becomes apparent at a certain depth of analysis:<sup>17</sup>

“Epistemically but not logically possible worlds [are] worlds so subtly inconsistent that the inconsistency could not be expected to be known (perceived) by an everyday logician, however competent.”

Mark Jago (2006) has pointed out that the urn models do not get to the heart of the problem. For example, urn models only take account of the depth of quantifiers, so on Hintikka’s revised theory agents are still represented as knowing all propositional tautologies and all propositional consequences of what they know. Moreover, different sentences may have the same quantifier depth yet be otherwise unrelated: use of Rantala’s models predicts that anyone knowing the one will also know the other, which seems simply false. They need not have what Stalnaker (1991, p. 435) calls the same accessibility to consciousness.

The case with signification is very similar to that with knowledge and belief. Just as we do not want to commit every knower to knowing every logical truth and every logical consequence of what they know, not even to knowing all propositional tautologies and tautologous consequences, so too we do not want a theory of meaning in which every sentence signifies all logical truths and all logical consequences of what it signifies. Nonetheless, in both cases, knowing, believing and signifying are closed under some limited

<sup>15</sup>Hintikka (1975, p. 475).

<sup>16</sup>Hintikka (1962, pp. 30 ff.).

<sup>17</sup>Hintikka (1975, p. 478).

or restricted notion of consequence, both wider and narrower than logical consequence. We have seen that it is narrower; but it is also wider. For example, if Socrates knows that Xantippe is at home, he knows his wife is at home, though that's not a logical consequence of what he knows but depends on the fact that Xantippe is his wife. Similarly, as we noted, Epimenides' unique utterance of 'All Cretans are liars' signifies not only that all Cretans are liars but also that Epimenides himself is a liar, since he was a Cretan—and again, that is not a logical consequence but what we might call a matter-of-fact or material consequence.

Jago notes that Cresswell (1973) and Priest (2002) do provide an alternative semantics for epistemic logic, employing non-normal worlds, which does not entail logical omniscience. However, their non-normal worlds are so “anarchic”, as Priest (2002, p. 459) terms it, that, as he points out, “it destroys all inferences concerning knowledge.” Although Priest admits (*loc.cit.*) that “knowledge is not that anarchic”, the only solutions he offers are all (logical omniscience) or none (anarchy), or an invitation to impose our own chosen constraint on non-normal worlds. Fine; but that is the challenge, not a solution.

#### 4. Indirect Speech Reports

The notion we seek, then, is a restricted concept of consequence which embraces the material consequences whereby, e.g., Epimenides' utterance signifies that Epimenides is a liar, but does not signify just any consequences whatever, such as that Epimenides was a Cretan, or Fermat's last theorem. The key lies, I suggest, in Bradwardine's first Conclusion. Bradwardine's argument given earlier was in support of his second Conclusion, that every sentence which signifies its own falsehood signifies its own truth and is false. Bradwardine's first Conclusion states:<sup>18</sup>

“Every sentence whose extreme has many supposita signifies or means affirmation or denial for some of them and if one, for it.”

‘Extreme’ was Aristotle's expression for the terms of a subject-predicate (*aka* categorical) sentence; and supposition is the medievals' own invention, a technical term whereby signification and supposition correspond very roughly to the modern concepts of meaning and reference. Many medieval writers equate ‘*x* supposits for *y*’ with ‘*x* stands for *y*’. In the proof of his first Conclusion, Bradwardine observes that, given a term *A*, *A* and ‘one of these’, pointing to the supposita of *A*, are equivalent as a matter of fact (*ut nunc*). We see this equivalence exemplified in the theory of supposition in the notion of “descent to singulars”. For example, William Ockham (1974, pp. 210-11) writes:

“Determinate supposition occurs when it is possible to descend to singulars by way of a disjunctive sentence. Thus, the following is a

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<sup>18</sup>Bradwardine (2010, ¶6.4, p. 97).

good inference: A man runs; therefore, this man runs or that man (and so on for singulars) . . .

“Merely confused supposition occurs when . . . it is not possible to descend to singulars by way of a disjunctive sentence . . . but it is by way of a sentence with a disjunctive predicate and it is possible to infer the original sentence from any singular. For example, . . . the following is a good inference: Every man is an animal; therefore, every man is this animal or that (and so on for singulars). The consequent here is a subject-predicate sentence composed of the subject ‘man’ and the predicate ‘this animal or that or that (and so on for singulars)’. It is clear that the predicate is truly predicated of every man and, therefore, the universal sentence is true.”

Thus ‘animal’ and ‘this animal or that animal or . . .’ are equivalent, that is, ‘one of the animals’. We see this also in Bradwardine’s application of his first Conclusion. He writes, e.g.,<sup>19</sup>

“These are equivalent as a matter of fact (*convertuntur ut nunc*): ‘A man is sitting’ and ‘A man is this thing sitting or that and so on for things sitting’.”

Again, “suppose that in some share-out all and only those who do not utter a falsehood will receive a penny, and suppose Socrates says: ‘Socrates will not receive a penny’, call it *A*.”<sup>20</sup>

“It should be said that Socrates will not receive a penny. To see this, it needs to be realised that ‘will not receive a penny’ and ‘utters a falsehood’ are equivalent as a matter of fact, because they are universally true of each other. Now *A* signifies that Socrates will not receive a penny, from which it follows as a matter of fact that Socrates utters a falsehood and from this it follows that what was said by Socrates is a falsehood and the subject of this conclusion has just one suppositum, namely *A*, so by the first Conclusion, it signifies that *A* is false, and so by the second postulate, *A* signifies itself to be false.”

Quine (1956, p. 177) famously distinguished a “notional” sense of intentional verbs from a “relational sense”: Ralph may believe quite generally that someone is a spy (on the evidence of a series of leaks) or quite specifically of someone (e.g., the man in the brown hat) that he is a spy (having seen him slip an envelope into a secret location). Traditionally, this would be marked as a scope distinction, or a *de re/de dicto* one, and language can be regimented to reflect it. But this is artificial, whether in Latin or English or in formal languages. In fact, as Priest (2002, p. 461) observes, talk *de dicto* collapses into *de re* as soon as proper names and other rigid designators are present.

<sup>19</sup>Bradwardine (2010, ¶ad 6.54, p. 101).

<sup>20</sup>Bradwardine (2010, ¶8.1, p. 125).



In my view, all such sentences generate an implicature, but nothing in the syntax determines how they should be understood. Rather, there is a Gricean conversational implicature that the term used indicates the agent's intention. E.g., if I report that Ralph believes Orcutt is a spy, or that the man in the brown hat is a spy, there is an implicature that Ralph knows his name, or saw him wearing a brown hat. But that inference is cancellable—we can add 'but he does not know his name', or 'he doesn't realise who the spy is', which shows that it is only an implicature, and not entailed by the sentence. There is no contradiction in saying 'Ernest is hunting rhinoceroses, but he thinks they're unicorns'. To be sure, to say 'There's a sloop he wants' "gives the wrong idea" (as Quine, 1956, p. 101 put it) if he's never heard of sloops, but again there is no contradiction once it is explained that he's looking for a boat for a certain purpose, and the speaker realises that his needs will be satisfied by a sloop. That such implications are cancellable, and non-detachable (that is, apply regardless of the particular expressions used), was for Grice the mark of conversational implicatures.

The constructions 'Socrates said that *p*', 'Ralph believes that *p*', 'Epimenides knows that *p*' and '*A* signifies that *p*' all permit substitution of co-referring terms in '*p*'. Such substitutions may give rise to puzzling implicatures, but they can be cancelled and explained. They are nonetheless valid. So 'St Paul said that Epimenides was a liar' is true even if St Paul did not realise that is what he had said when he reported "One of themselves, a prophet of their own, said 'Cretans are always liars'."<sup>21</sup> Again, you know the one approaching, since he is Coriscus and you know him.<sup>22</sup> It's misleading to say so, since there is an implicature that you have recognised him, despite his wearing a hood. But the implicature is cancellable. Similarly, as we noted, in Cappelen and Lepore's example, we can correctly report that Jack said that a French woman bought the picture, even if he did not realise she was French.

The construction '*A* signifies that *p*' can be replaced by '*A* means that *p*' and '*A* says that *p*', and anyone uttering *A* says what *A* says, means and signifies. If we report that such and such a sentence says that Epimenides is a liar, we might infer that the sentence is 'Epimenides is a liar'. But it might also be 'All Cretans are liars', or 'Κρηῆτες ἀεὶ ψεῦστα'. 'All men are animals' signifies that every man is one of the animals, this animal or that animal or another. 'Socrates utters a falsehood' signifies that one of Socrates' utterances is false and so, if that was Socrates' only utterance, it signifies of itself that it is false.

Kripke's example (1979) of Pierre, marooned in an ugly part of London, may be thought to disprove the intersubstitutivity claim. The example is extremely contrived and unnatural, but it is not wrong to report Pierre as believing both that London is pretty (since as a French speaker he would assent to 'Londres est joli') and that London is not pretty (since he believes

<sup>21</sup> *Titus* 1:12 (King James version).

<sup>22</sup> Aristotle, *De Sophisticis Elenchis*, 179a34.

the place where he is living to be ugly, and it is indeed, unknown to him, London). He has contradictory beliefs, though he does not realise it. Any implicature, that he recognises the contradiction, or that he would assent to ‘London is not pretty’—or for that matter, to ‘London is pretty’—is certainly misleading, but can be cancelled.

## 5. Bradwardine’s Second Conclusion

We may conclude, therefore, that intentional reports are closed under co-reference of their terms. Bradwardine appeals to this principle in his use of his first Conclusion in his argument, given in §1 above, for his second Conclusion. Recall that he first showed that no sentence can signify only that it itself is false. So suppose  $A$  signifies that  $A$  is false and that  $\phi$ —meaning is multiple, letting  $\phi$  embrace everything else  $A$  signifies. Then if  $A$  is false, things are not wholly as it says they are, so either not- $\phi$  or  $A$  is not false but true, using the first Conclusion to replace ‘wholly as it says they are’ by ‘ $A$  is false and  $\phi$ ’, using the fourth postulate (the so-called De Morgan laws) to infer ‘either not- $\phi$  or  $A$  is not false’ from ‘not- $(A$  is false and  $\phi)$ ’, and using the first postulate (Bivalence) to infer ‘ $A$  is true’ from ‘ $A$  is not false’. But  $A$  signifies (*inter alia*) that  $A$  is false. So by the closure postulate,  $A$  signifies that either not- $\phi$  or  $A$  is true. Moreover,  $A$  signifies that  $\phi$ , and by Disjunctive Syllogism,<sup>23</sup> if not- $\phi$  or  $A$  is true, and  $\phi$ , then  $A$  is true, so again by the closure postulate,  $A$  signifies that  $A$  is true. Thus any sentence which signifies, among other things, that it itself is false, also signifies itself to be true.

It is tempting, therefore, to impose this restricted notion of consequence for use in Bradwardine’s second postulate, namely, that signification is closed under substitution of co-referring terms, as given by Bradwardine’s first Conclusion. However, Bradwardine’s argument also appeals to his other postulates, Bivalence and Contravalance (every sentence is either true or false and not both—his first postulate), the De Morgan laws and Disjunctive Syllogism, and he also appeals to the lattice laws for conjunction and disjunction (the sixth postulate), in extending his solution to disjunctive and conjunctive versions of Curry’s paradox.<sup>24</sup> These include all the ingredients for the notorious derivation of *ad impossibile quodlibet* (from an impossibility anything follows) by what is often called “Lewis’ argument” (referring to C.I. Lewis) but was also known to the twelfth-century logicians of the School of Adam Balsham (the Adamites). The argument shows that anything follows from a formal impossibility of the form ‘ $p$  and not- $p$ ’:

<sup>23</sup>His fifth postulate (Bradwardine, 2010, ¶6.3, p. 97): “From any disjunction together with the opposite of one of its parts the other part may be inferred.”

<sup>24</sup>See, e.g., Bradwardine (2010, ¶¶8.4-8.5, p. 129).

Suppose	$p$ and not- $p$	is true	
Then	$p$	is true	by the sixth postulate
So	$p$ or $q$	is true	by the sixth postulate
But	not- $p$	is true	by the sixth postulate
So	$q$	is true	by the fifth postulate

The sixth postulate entails Simplification and Addition, used here in lines 2-4. It reads:<sup>25</sup>

“If a conjunction is true each part is true and conversely; and if it is false, one of its parts is false and conversely. And if a disjunction is true, one of its parts is true and conversely; and if it is false, each part is false and conversely.”

Since any sentence signifying its own falsehood signifies its own truth, by the second Conclusion, it signifies contradictory things, and so by *ad impossibile quodlibet* and the second postulate, it signifies everything.

More generally, Walter Burley, in his *Treatise on Consequence* of 1302, which Bradwardine would very likely have known, showed that anything follows from an impossibility (formal or not) and any necessity follows from anything (*necessarium sequitur ad quodlibet*) by the topic “from the lesser” (*per locum a minori*):<sup>26</sup>

“From the impossible anything follows by the topic *from the lesser*. For if it is argued in this way: a man is an ass, so you are running, this inference holds by this rule, namely, if what seems to belong less belongs, then what seems to belong more will belong ; but that ass belongs to man seems less than that running belongs to man.

“Similarly, what is necessary follows from anything by the topic *from the lesser*. Hence it follows: you are running, so there is a God. And it holds by this rule, namely the one given above, for that running belongs to man seems less than that being belongs to God.”

Consequently, if we accept the proposal that every sentence signifies or means whatever follows from what it signifies, every sentence will signify every necessity and any contradictory sentence will signify everything. Indeed, since every truth follows as a matter-of-fact consequence from anything, and anything whatever follows in a matter-of-fact consequence from what is false (*ex*

<sup>25</sup>Bradwardine (2010, ¶6.4, p. 97).

<sup>26</sup>Green-Pedersen (1980, p. 129): *Item: ex impossibili sequitur quodlibet per locum a minori. Quia si sic arguatur ‘homo est asinus, ergo tu curris,’ ista consequentia tenet per regulam istam, scilicet si illud quod minus videtur inesse inest, et illud quod magis videtur inesse inest; sed minus videtur inesse quod asinus insit homini quam cursus insit homini. Similiter: necessarium sequitur ad quodlibet per locum a minori. Unde sequitur ‘tu curris, ergo deus est.’ Et tenet per hanc regulam, scilicet prius datam, quia minus videtur quod cursus insit homini quam esse insit deo. Cf. Burley (1955, p. 248); Burley (2000, p. 62).*

*falso quodlibet*), every sentence would accordingly signify every truth, and every sentence signifying what is false would signify everything.<sup>27</sup> This would trivialize Bradwardine's account of signification.

So let us we reject closure by the second postulate under consequence in general, and restrict it to closure under consequence solely by the first Conclusion, that is, under co-reference, as we saw was common practice in indirect speech reports. What of the second Conclusion, on which Bradwardine's solution to the semantic paradoxes depends? Bradwardine's own proof of the second Conclusion now fails to go through. Does Bradwardine's solution then collapse?

No: the proof I sketched in §1 does not depend on the fifth and sixth postulates, but only on the principle of Importation:

(Imp) 'if  $\phi$  then  $\psi$ ' follows from  $\Pi$  only if  $\psi$  follows from  $\Pi$  and  $\phi$

So let  $A$  signify that  $A$  is false and that  $\phi$ —where  $\phi$  embraces everything else  $A$  signifies. Then if  $A$  is false, things are not wholly as it says they are, so either not- $\phi$  or  $A$  is true, by the first Conclusion and the first and fourth postulates. That is, if  $A$  is false, then if  $\phi$  then  $A$  is true. So by (Imp), if  $A$  is false and  $\phi$ , then  $A$  is true. But  $A$  signifies that  $A$  is false and  $\phi$ , so by the closure postulate,  $A$  signifies that  $A$  is true. Thus any sentence which signifies, among other things, that it itself is false, also signifies itself to be true.

(Imp) is a fairly modest principle. It holds, e.g., in relevance logic with an intensional implication and an extensional conjunction (unlike its converse, (Exp), which only holds for a detachable implication if the conjunction is intensional).<sup>28</sup> So too do the first and fourth postulates—Bivalence, Contravalence and the De Morgan principles. If the implication ('if  $\phi$  then  $A$  is true') in the proof of the second Conclusion is intensional, so too is the disjunction from which it is inferred, but that is plausible, i.e., if  $A$  signifies that  $A$  is false and  $\phi$ , then if things are not wholly as  $A$  signifies then if  $\phi$  then  $A$  is not false but true.

In point of fact, there is an even more direct proof of the second Conclusion, indeed, of the stronger result that every sentence signifies its own truth. The claim that every sentence signifies its own truth was made by Bonaventure, Scotus, Burley, the early Buridan and Albert of Saxony—*pace* Buridan, I'll call it "Buridan's Principle".<sup>29</sup> Here is a proof mimicking just the method of proof used by Bradwardine for his second Conclusion:

Suppose  $A$  signifies that  $\phi_1, \phi_2$  and so on. Let  $\phi$  be their conjunction—everything  $A$  signifies. Then by Bradwardine's account of truth,  $A$  is true if and only if things are wholly as  $A$  signifies, that is, by the first Conclusion, if and only if  $\phi$ . In particular, if  $\phi$  then  $A$  is true. But  $A$  signifies that  $\phi$ . So by the second postulate,  $A$  signifies that  $A$  is true. Thus, every sentence

<sup>27</sup>See, e.g., Restall (2008, pp. 229-30).

<sup>28</sup>See, e.g., Read (2007, p. 252).

<sup>29</sup>See, e.g., Buridan (1994, p. 94). He rejected the principle in later works. See, e.g., Buridan (2001, *Sophismata* ch. 8, p. 967).

signifies its own truth. Hence any sentence which says of itself that it is false, signifies both that it is true and that it is false. It cannot be both, so things cannot be wholly as it says they are, so it is false.

Hence, Bradwardine's solution does not depend on principles which would commit one to *ad impossibile quodlibet* and hence to an essentially trivial account of signification via the second postulate. One certainly needs to close signification under more than just substitution of co-referents, as permitted by the first Conclusion. For example, to apply Bradwardine's second Conclusion to Epimenides' paradox, we need to infer that Epimenides' utterance of 'All Cretans are liars' says of itself that it is false, and that inference depends on Universal Instantiation (as well as the fact that Epimenides was a Cretan).

Or take the validity paradox, recently dubbed V-Curry.<sup>30</sup> V-Curry is the inference from 'V-Curry is valid' to an arbitrary conclusion,  $p$ . The premise signifies that V-Curry is valid—but it may signify more, call it  $\phi$ . Now suppose V-Curry is valid and  $\phi$ . Then things are as V-Curry's premise signifies, so V-Curry is a valid inference with true premise, so things are as the conclusion signifies, i.e.,  $p$ . But V-Curry's premise signifies that V-Curry is valid and  $\phi$ , so it signifies that  $p$ , by the closure principle. Consequently, if things are as V-Curry's premise signifies then they are as the conclusion signifies. So V-Curry is valid.<sup>31</sup> However, we cannot infer that  $p$ , unless things are wholly as V-Curry's premise signifies, that is, unless  $p$ . The application of the closure principle here depends only on closure under detachment, as well as the first Conclusion.

So only modest principles are needed to apply Bradwardine's solution, certainly nothing so strong as to trivialize it. Indeed, this collection of principles is the same as that found in the logic of knowledge, and in the logic of indirect speech reports. At their heart is the principle of substitution recorded in Bradwardine's first Conclusion, supplemented by the quantifier rules, detachment, and the analytic consequences of a relevance logic. Whether or not one accepts the validity of Disjunctive Syllogism, signification need not be closed under it for Bradwardine's solution to be applied.

## 6. Conclusion

Bradwardine's solution to the logical paradoxes depends on three claims:

- the multiplicity of meaning, that every proposition signifies many things
- that any proposition's truth depends on things' being wholly as it signifies
- that signification is closed under consequence, that is, that a proposition signifies everything that follows from anything it signifies.

<sup>30</sup>See, e.g., Read (2010), Beall and Murzi (2013). Actually, for Beall and Murzi, 'v-Curry' denotes the premise of an instance of the inference here called 'V-Curry', with conclusion ' $\perp$ ', that is, absurdity.

<sup>31</sup>A fuller account would examine the concept of validity more closely here, to check that this inference holds.

The first of these claims echoes claims made recently for semantic pluralism, *aka* propositional profusion. The second is a natural revision of the truth-equivalence, in the light of endorsement of the first claim. The third claim is the most controversial, similar to claims for logical omniscience and sharing their implausibility. Bradwardine's practice suggests that he often appeals only to closure under his first Conclusion, that is, substitution of co-referring terms. However, substitution under co-reference is insufficient for Bradwardine's own proof of his second Conclusion to go through, thus undermining the whole basis of his solution. Arguably, the real problem lies with the fifth postulate, Disjunctive Syllogism, which need not be used since his proof can be revised to turn on the principle of Importation. Bradwardine's own principles, indeed, allow a simpler proof to be given showing that every sentence signifies its own truth. All in all, the closure postulate need only apply closure to modest principles, sufficient to show that the paradoxical sentences are in fact self-contradictory, without trivializing Bradwardine's account of signification. Hence, solving the semantic paradoxes does not require revision of logic, thus saving logic from paradox.

## References

- Beall, J. and Murzi, J. (2013). Two flavors of Curry's paradox. *Journal of Philosophy*, 110:143–65.
- Bradwardine, T. (2010). *Insolubilia*. Peeters, Leuven. Edited with English translation by S. Read.
- Buridan, J. (1994). *Questiones Elencorum*. Ingenium, Nijmegen. Ed. R. van der Lecq and H.A.G. Braakhuis.
- Buridan, J. (2001). *Summulae de Dialectica*. Yale U.P., New Haven. English translation by G. Klima.
- Burley, W. (1955). *De Puritate Artis Logicae Tractatus Longior, with a revised edition of the Tractatus Brevior*. The Franciscan Institute, St Bonaventure.
- Burley, W. (2000). *On the Purity of the Art of Logic*. Yale UP, New Haven and London. English translation by P.V. Spade.
- Cappelen, H. and Lepore, E. (2005). *Insensitive Semantics: A Defense of Semantic Minimalism and Speech Act Pluralism*. Blackwell Pub.
- Chalmers, D. (2010). *The Character of Consciousness*. Oxford UP, Oxford.
- Cresswell, M. (1973). *Logics and Languages*. Methuen, London.
- Dorr, C. (2011). Propositional profusion and the Liar. Unpublished typescript: <http://philpapers.org/rec/DORPPA-2>.
- Green-Pedersen, N. J. (1980). Walter Burley's *De consequentiis*: an edition. *Franciscan Studies*, 40:102–166.
- Grice, H. P. (1975). Logic and conversation. *Syntax and Semantics*, 3:41–58. Reprinted in Grice (1989), pp. 22–40.
- Grice, H. P. (1989). *Studies in the Way of Words*. Harvard University Press, Cambridge, Mass.

- Hawthorne, J. and Dorr, C. (2011). Semantic plasticity. To appear.
- Hintikka, J. (1962). *Knowledge and Belief*. Cornell University Press, Ithaca.
- Hintikka, J. (1975). Impossible possible worlds vindicated. *Journal of Philosophical Logic*, 54:475–84.
- Jago, M. (2006). Hintikka and Cresswell on logical omniscience. *Logic and Logical Philosophy*, 15:325–54.
- Kripke, S. (1979). A puzzle about belief. In Margalit, A., editor, *Meaning and Use*, pages 239–93. Reidel, Dordrecht. Reprinted in Kripke (2011), pp. 125–61.
- Kripke, S. (2011). *Philosophical Troubles*. Oxford University Press, Oxford. Collected Papers vol. 1.
- Martin, R. L., editor (1984). *Recent Essays on Truth and the Liar Paradox*. Clarendon Press, Oxford.
- Ockham, G. de. (1974). *Summa Logicae*. Franciscan Institute Publications, St Bonaventure. Edited by P. Boehner et al.
- Priest, G. (2002). The hooded man. *Journal of Philosophical Logic*, 31:445–67.
- Prior, A. (1958). Epimenides the Cretan. *Journal of Symbolic Logic*, 23:261–66.
- Quine, W. (1956). Quantifiers and propositional attitudes. *Journal of Philosophy*, 53:177–87.
- Read, S. (2007). Bradwardine’s revenge. In Beall, J., editor, *Revenge of the Liar*, pages 250–261. Oxford UP, Oxford.
- Read, S. (2010). The validity paradox. In Peliš, M., editor, *The Logica Yearbook 2009*, pages 209–21. College Publications, London.
- Restall, G. (2008). Modal models for Bradwardine’s theory of truth. *Review of Symbolic Logic*, 1:225–40.
- Soames, S. (2002). *Beyond Rigidity: the Unfinished Semantic Agenda of Naming and Necessity*. Oxford University Press, New York.
- Soames, S. (2008). The gap between meaning and assertion: why what we literally say often differs from what our words literally mean. In *Philosophical Essays*, pages 251–77. Princeton UP, Princeton.
- Stalnaker, R. (1991). The problem of logical omniscience, I. *Synthese*, 89:425–40.

Stephen Read  
 University of St Andrews  
 Arché Research Centre  
 St Andrews KY16 9AL  
 Scotland.  
 e-mail: slr@st-and.ac.uk