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John Buridan's Theory of Consequence and his Octagons of Opposition*

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Abstract

One of the manuscripts of Buridan's *Summulae* contains three figures, each in the form of an octagon. At each node of each octagon there are nine propositions. Buridan uses the figures to illustrate his doctrine of the syllogism, revising Aristotle's theory of the modal syllogism and adding theories of syllogisms with propositions containing oblique terms (such as 'man's donkey') and with propositions of "non-normal construction" (where the predicate precedes the copula). O-propositions of non-normal construction (i.e., 'Some S (some) P is not') allow Buridan to extend and systematize the theory of the assertoric (i.e., non-modal) syllogism. Buridan points to a revealing analogy between the three figures. To understand their importance we need to rehearse the medieval theories of signification, supposition, truth and consequence.

John Buridan was born in the late 1290s near Béthune in Picardy. He studied in Paris and taught there as Master of Arts from the mid-1320s, and wrote his *Treatise on Consequences* sometime after 1335. He composed his *Summulae de Dialectica* (*Compendium of Dialectic*) from the late 1330s onwards, in eight (or nine) treatises with successive revisions into the 1350s. His *Sophismata*, the ninth treatise of the *Summulae* (sometimes presented as a separate work) has been much studied in the last fifty years. He wrote many other works, mostly commentaries on Aristotle and died around 1360.

Three intriguing figures occur in one of the manuscripts of Buridan's *Summulae*. They are Octagons. In addition, there is a particularly elaborate Square of Opposition. See Figure I.[1] In order to understand what doctrines

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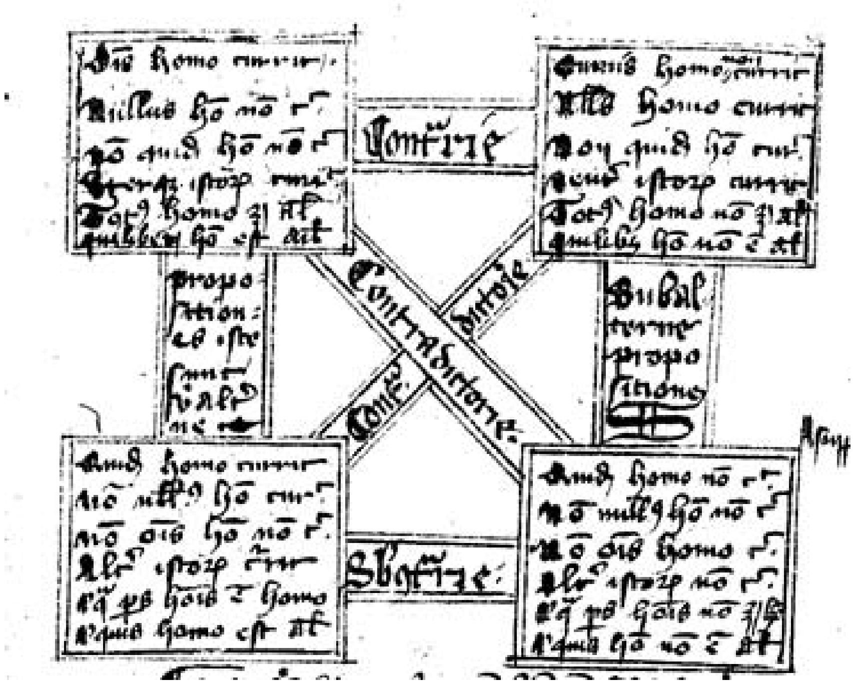


Figure 1: Buridan's Square of Opposition

Buridan is setting out in these figures, we need first to fill in some background in Aristotle's theory of the syllogism and in medieval theories of signification, supposition and consequence.

1 The Assertoric Syllogism

For Aristotle, according to Buridan, a syllogism consists of two premises, a major premise and a minor premise, made up of three terms. The major premise, containing the major term, is the first premise. The middle term appears in both premises. Aristotle's goal is to identify which pairs of premises, that is, figures, yield a conclusion:

> "It seems to me that Aristotle takes a syllogism not to be composed of premises and conclusion, but composed only of premises from which a conclusion can be inferred." (Hubien, 1976, Book III ch. 4, p. 92)

Hence there are three figures:

Figure 1 where the middle term is subject of one premise and predicate of the other;

Figure 2 where the middle term is predicate of both premises; and

Figure 3 where the middle term is subject in both premises

Each proposition (or declarative sentence) is of one of four forms, *a*-, *e*-, *i*- and *o*-propositions, forming the traditional Square of Opposition. Validity of the Aristotelian syllogism is based on the *dictum de omni et nullo*:

> "We say that one term is predicated of all of another when no examples of the subject can be found of which the other term cannot be asserted. In the same way, we say that one term is predicated of none of another." (*Prior Analytics* I 1, 24a28-30)

These establish the validity of the direct moods in the first figure: Barbara, Celarent, Darii and Ferio. The validity of all remaining syllogisms can be reduced to the direct moods in the first figure by conversion or reduction *per impossibile*:[2]

- Simple conversion: AiB implies BiA and AeB implies BeA
- Conversion *per accidens*: AaB implies BiA[3]
- Reduction *per impossibile*: using the first-figure syllogisms to show that the premises are incompatible with the contradictory of a putative conclusion.

2 Aristotle's Theory of the Modal Syllogism

Aristotle extended his theory of the syllogism to include the modal operators 'necessarily', 'contingently' and 'possibly'. He endorsed the K-principles:

> "If, for example, one should indicate the premises by A and the conclusion by B, it not only follows that if A is necessary B is necessary, but also that if A is possible, B is possible." (*Prior Analytics* I 15, 34a22-24)

that is:

- from $\Box(A \to B)$ infer $\Box A \to \Box B$, and
- from $\Box(A \to B)$ infer $\Diamond A \to \Diamond B$

[2]The names of the moods in the medieval mnemonic record the reduction procedure:

> Barbara Celarent Darii Ferio Baralipton
> Celantes Dabitis Fapesmo Frisesomorum;
> Cesare Camestres Festino Baroco; Darapti
> Felapton Disamis Datisi Bocardo Ferison.

[3]We can also convert AeB *per accidens* to BoA, although Aristotle does not call this a conversion (since AeB converts to BeA). But BeA implies its subaltern BoA, for Aristotle writes at 26b15: "'P does not belong to some S' is ... true whether P applies to no S or does not belong to every S."

Aristotle also explicitly accepts the characteristic thesis of necessity: $\Box A \to A$ ("that which is of necessity is actual"—*De Interpretatione* 13, 23a21), and the thesis relating necessity and possibility: $\neg\Diamond A \leftrightarrow \Box\neg A$ ("when it is impossible that a thing should be, it is necessary ... that it should not be" and vice versa—22b5-6).

Mixed syllogisms allow premises with different modality. There are two Barbaras with one modal premise:

Necessarily all M are P	All M are P
All S are M	Necessarily all S are M
So necessarily all S are P	So necessarily all S are P

In *Prior Analytics* I 9, Aristotle says the first, Barbara LXL, is valid, but the second, Barbara XLL, is not. The natural interpretation (adopted by the medievals, and probably due to Peter Abelard) is that he reads 'Necessarily all M are P' *de re*, that is, as 'All M are necessarily P'. But that conflicts with I 3, where he says that 'Necessarily all M are P' converts simply to 'Necessarily some P are M', which seems only to be valid *de dicto*. So a major problem for the medievals was to try to produce a coherent account of the modal syllogism.

3 Signification

In *De Interpretatione* 1, Aristotle distinguished three kinds of terms, written, spoken and mental:

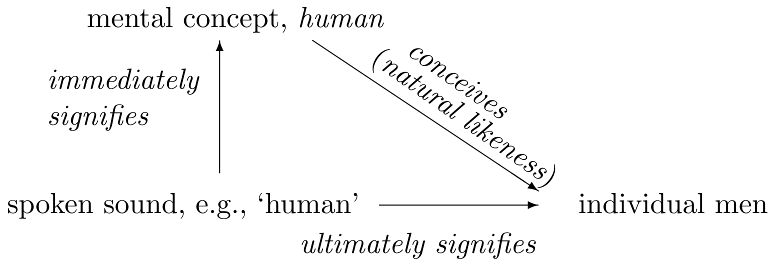
> "Spoken [words] are signs (*notae*) of impressions (*passionum*) in the soul, and written of spoken." (16a3-4)

Boethius glossed this as: "thing, concept, sound and letter are four: the concept conceives the thing, spoken sounds are signs of the concept, and letters signify the sounds."[4] But according to Augustine, words signify things, by means of the concepts. The medievals were left to ask: do words signify concepts or things? On the one hand, if words signified things, empty names would be impossible. But on the other, when I say Socrates runs, I mean that Socrates, not some concept of Socrates runs. The thirteenth century insight was that the concept is itself a sign. In the words of Buridan:

> "Categorematic words ... signify things by the mediation of their concepts, according to which concepts, or similar ones, they were imposed to signify. So we call the things conceived by those concepts 'ultimate *significata*' ... but the concepts we call 'immediate *significata*'." (Buridan, 2001, IV 3, p. 253-4)

[4]Boethius (1877, p. 37).

We can picture Buridan's theory of signification as follows:



But what of the signification of propositions? Abelard had spoken of the *dictum* of a proposition, what it signified. We find this idea in the anonymous *Ars Burana*, dating from the 12^{th} C.:

> "We speak about the *dictum* of a proposition or of the significate of the proposition or of an *enuntiabile* ... For example: 'A human is an animal' is true because what it signifies is true, and that true thing is the *enuntiabile* ... Some enuntiabilia are of the present, some are of the past and some are of the future." (Anonymous, 1967, p. 208)

In the 14^{th} C., Ockham tried to explain everything without recourse to such entities. What is the object of scientific knowledge? It was the focus of the London debate in the 1320s between the Franciscans, William Ockham, Walter Chatton and Adam Wodeham. Ockham put forward his "Razor": "No plurality should be assumed unless it can be proved by reason, or experience, or by some infallible authority."[5] Accordingly, Ockham says that the object of knowledge is the mental proposition (*complexum*) that is, the *actus sciendi* is sufficient on its own, just as the *actus intellectus* is the only universal he recognises. Chatton countered with his "Anti-Razor": if n things are not enough to explain something, posit $n + 1$. So Chatton claimed that the object of knowledge was the thing or things signified by the terms in the proposition.[6] Adam Wodeham's *via media* (1331) introduced the notion of the *complexe significabile*, things' being a certain way: the object of knowledge is the state of affairs (*modus rei*) signified by the proposition, the "complexly signifiable" (*complexe significabile*) that can only be signified in a complex way, that is, by a proposition.[7]

The theory came to Paris through Gregory of Rimini (1344): we must recognise *enuntiabilia* or *complexe significabilia* as things: "every complex or incomplex signifiable, whether true or false, is said to be a thing (*res*) and to be something (*aliquid*)."[8] Buridan rejected all talk of *complexe significabilia*. It turns on taking truth to be a matter of signification, when it is really one of supposition:

[5] See, e.g., Adams (1987, p. 1008).
[6] See, e.g., Keele (2006, p. 25).
[7] See Zupko (1997).
[8] Cited in Zupko (1997, p. 221).

> "If we can explain everything by fewer, we should not ... posit many, because it is pointless to do with many what can be done with fewer. Now everything can be easily explained without positing such *complexe significabilia*, which are not substances, nor accidents, nor subsistent *per se*, nor inherent in anything else. Therefore, they should not be posited."[9]

4 Truth and Consequence

Buridan states his theory of truth in terms of supposition, not signification. First, we need to recall the main divisions of supposition:

- Material supposition: that of a term standing for itself or a similar expression, which it was not imposed to signify[10]
- Personal supposition: that of a term standing for the singulars of which it is predicated

In addition, there are various modes of personal supposition:

- Discrete supposition: that of a discrete, or singular term
- Common supposition, that of a common term, defined by the terminists using the notions of ascent and descent:
  - Determinate supposition: when one can descend to a disjunction of singulars, and ascend from any singular, with respect to that term; otherwise:
  - Confused and distributive supposition: if one can descend to an indefinite conjunction of singulars with respect to that term; otherwise:
  - Merely confused supposition (allowing descent to a disjunct term).

Thus, e.g., 'Some A is not B' is equivalent to 'This A is not B or that A is not B and so on for all the As' (i.e., $\bigvee_i A_i$ is not B, so 'A' has determinate supposition) and to 'Some A is not this B and some A is not that B and so on for all the Bs' (i.e., $\bigwedge_j$ Some A is not B_j, so 'B' has confused and distributive supposition)—where $\bigvee \emptyset$ is true and $\bigwedge \emptyset$ is false. Thus, e.g., 'Every A is B' is equivalent to 'This A is B and that A is B and so on for all the As' (i.e., $\bigwedge_i A_i$ is B, so 'A' has confused and distributive supposition) and to 'Every A is this B or that B and so on for all the Bs' (i.e., Every A is $\bigvee_j B_j$, so 'B' has merely confused supposition)—where $\bigvee \emptyset$ is false and $\bigwedge \emptyset$ is true.

[9] Buridan, *Commentary on Aristotle's Metaphysics*, cited in Zupko (1997, p. 224).

[10] Simple supposition, of a term for a universal or concept) was subsumed by Buridan under material supposition).

Moreover, some words have the effect of widening or narrowing the supposition of other terms in a proposition. The past tense ampliates the subject to include past as well as present supposita. For example, 'A white thing was black' means that something which is now white or was white in the past was black. The future tense ampliates the subject to include future as well as present supposita. Modal verbs ampliate the subject to possible supposita,[11] as do verbs such as 'understand', 'believe', and verbal nouns ending in '-ble': 'possible', 'audible', 'credible', 'capable of laughter'. Other expressions restrict the supposition of terms, e.g., qualifying 'horse' with the adjective 'white' restricts the supposition of 'horse' in 'A white horse is running' to white horses.

Buridan notes that some say a proposition is true if things are as it signifies they are (*qualitercumque ipsa significat ita est*).[12] He objects: if Colin's horse cantered well, 'Colin's horse cantered well' is true, but his horse is now dead, so nothing at all is signified. So perhaps a proposition is true if things are, were or will be as it signifies they are, were or will be. But, given his rejection of *complexe significabilia*, 'A human is a donkey' signifies humans and donkeys, which are indeed humans and donkeys, but is not true.[13] The moral is that to assign truth and falsity, we have to go beyond signification and consider supposition. A particular affirmative is true if subject and predicate supposit for the same; a particular negative is true if subject and predicate do not supposit for the same things; a universal affirmative is true if the predicate supposits for everything the subject supposits for; and so on.[14]

Turning to the theory of consequence, Buridan notes that some say a consequence is valid if the premise cannot be true without the conclusion's being true.[15] He objects: 'Every human runs, so some human runs' is valid, but it's possible for the premise to be true without the conclusion even existing, and so without it being true. We might therefore say that a consequence is valid if the premise cannot be true without the conclusion's being true, when both are formed together. But 'No proposition is negative, so no donkey is running' is not valid (for its contrapositive is not valid), but the premise cannot be true, and so cannot be true without the conclusion's being true. Finally, we could say that a consequence is valid if it is impossible that however the premise signifies to be it is not however the conclusion signifies, when they are formed together Buridan accepts this, provided that 'however it signifies' is not taken literally, but in the sense previously given.

Buridan contrasts formal and material consequence, but gives them an unusually modern gloss:

> "A consequence is called formal if it is valid in all terms retain-

[11] As Aristotle noted: *Prior Analytics* I 13, 32b25-28.
[12] Hubien (1976, I 1, p. 17).
[13] Buridan (2001, *Sophismata* 2, sophism 5, pp. 847-8).
[14] Buridan (2001, *Sophismata* 2, conclusion 14, p. 858).
[15] Hubien (1976, I 3, p. 21).

> ing a similar form ... A material consequence, however, is one where not every proposition similar in form would be a good consequence, or, as it commonly put, which does not hold in all terms retaining the same form; e.g., A human runs, so an animal runs, because it is not valid with these terms: A horse walks, so wood walks." (Hubien, 1976, I 4, p. 23)

Finally, he distinguishes absolute from *ut nunc* (or "as a matter of fact") consequence:

> "Some material consequences are called absolute consequences (*consequentiae simplices*) because they are good consequences absolutely, since it is not possible for the antecedent to be true the consequent being false. Others, which are not good absolutely, are called as-of-now consequences (*consequentiae ut nunc*) when it is possible for the antecedent to be true without the consequent, but are good as-of-now, when things being as a matter of fact as they are, it is impossible for the antecedent to be true without the consequent." (Hubien, 1976, I 4, p. 23)

So much for background. We can now consider what doctrines Buridan is illustrating in his figures.

5 Non-normal Form

Besides the further complication of the octagons, even the simple Square of Opposition in Pal.lat. 994 is more complex than usual, in containing several propositions at each node. (See Figure 1.) The set of propositions at each node are equivalents, or at least of the same type. For example, at the top left node we read:

- 'Every human is running' (*Omnis homo currit*)
- 'No human is not running' (*Nullus homo non currit*)
- 'Not any human is not running' (*Non quidam homo non currit*)
- 'Each of these is running' (*Uterque istorum currit*)
- 'The whole [of] human is an animal' (*Totus homo est animal*)
- 'Any human is an animal' (*Quilibet homo est animal*)

In the octagons, each of the eight nodes contains nine equivalent propositions (or again, of the same type). For example, in the octagon of non-normal propositions (see Figure 2), we read:

- 'Every B every A is' (*Omne B omne A est*)

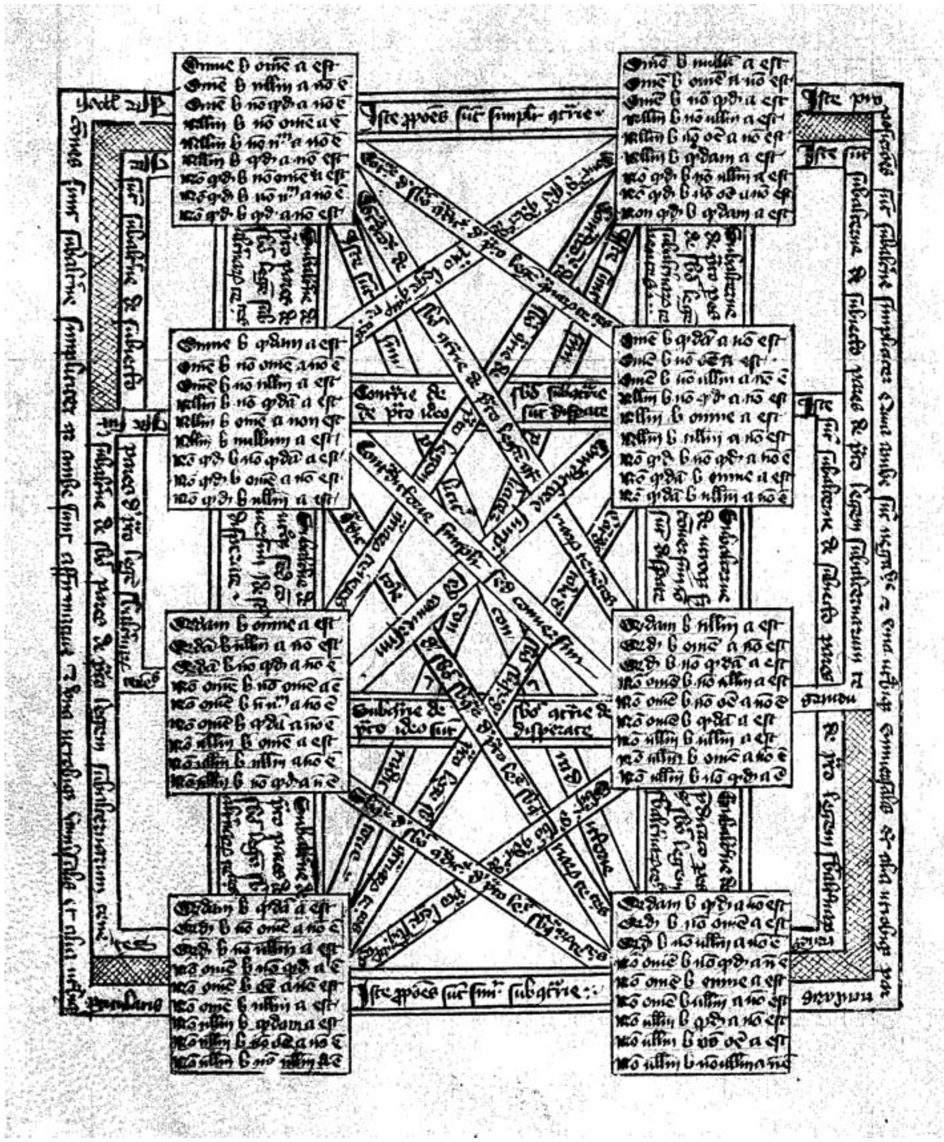


Figure 2: Buridan's Octagon of Opposition for Non-Normal Propositions

- 'Every B no A is not' (*Omne B nullum A non est*)
- 'No B not every A is' (*Nullum B non omne A est*)
- ...

This "non-normal construction" (or "unusual way of speaking"—*de modo loquendi inconsueto*) consists in placing the predicate before the copula. Placing the predicate before the copula often has the effect of changing the sense: e.g., 'Some animal a human is not' means, not that some animal is not a human, but that some animal is not some human:

> "I call it normal form where an external negation precedes the predicate, e.g., 'No B is A', 'Some B is not A'. But I call the form non-normal where the predicate precedes the negation and so is not distributed by it, e.g., 'Every B A is not', 'Some B A is not'." (Hubien, 1976, I 8, p. 44)

This is an example of the increasing use in late medieval logic of regimented vernacular forms. What Buridan here calls the non-normal form is in fact the normal form used by, e.g., Apuleius and Boethius.[16] What Buridan does is to interpret the so-called non-normal form non-normally, so that, for example, A in 'Every B A is not' and 'Some B A is not' is not distributed by the negation, and so does not have confused and distributive supposition. One can see it as a further regimentation similar to that made by Aristotle in *Prior Analytics* I 46 when he distinguished 'me einai leukon' ('is not white') from 'einai me leukon' ('is not-white').

Using non-normal propositions, Buridan is able to extend accidental conversion (i.e., conversion *per accidens*) from universal propositions to particular propositions. Then each categorical (or "predicative"—*propositio categorica*) assertoric proposition can be converted, either simply or accidentally:

- 'Every A is B' converts accidentally to 'Some B is A'
- 'No A is B' converts simply to 'No B is A' and accidentally to 'Some B is not A'
- 'Some A is B' converts simply to 'Some B is A'
- 'Some A is not B' converts accidentally to 'Every B A is not' (e.g., 'Some animal is not a human, so every human (some) animal is not'—*quoddam animal non est homo, igitur omnis homo animal non est*)

> "Accidental conversion involves changing the predicate into the subject and the subject into the predicate while preserving the quality but changing the quantity." (Buridan, 2001, I 6, p. 51)

Buridan deploys these ideas in his theory of the syllogism. In the fourteenth century, the syllogism was brought under a general theory of consequence. Buridan treats the syllogism as a special case of formal consequence. But he introduces a novel approach to demonstrating the validity of assertoric syllogisms. For example, he establishes the following theorems:

- No valid syllogism has two negative premises
- In every valid syllogism, the middle term is distributed (that is, has confused and distributive supposition) at least once

[16] See Londey and Johanson (1987) and Thomsen Thörnqvist (2008).

- A proposition with a distributed term entails the same proposition with the term undistributed, but never vice versa

These tests for validity were preserved in the traditional rules of the syllogism.[17] Then Buridan can go systematically through all combinations of premises and conclusion and identify the valid syllogisms.

These include syllogisms with "non-normal" propositions. Using such propositions, Buridan extends the assertoric syllogism to additional cases, so that there are eight valid moods in each of the first and second figures, and nine in the third.[18] Remember that here by 'mood', Buridan means 'combination of premises'. To the first figure (with the four perfect moods, Barbara, Celarent, Darii and Ferio), he adds:

- MiP, SoM: SPo (Some M is P, some S is not M, so some S some P is not)
- MaP, SoM: SPo (Every M is P, some S is not M, so some S some P is not)

and he counts Fapesmo (MaP, SeM: PoS) and Frisesomorum (MiP, SeM: PoS) as making up the eight (direct and indirect) valid moods in the first figure. Thus, e.g., Baralipton (in the mnemonic) and Barbari (not in the mnemonic) do not count for Buridan as additional to Barbara. So the order of the premises counts for Buridan, but whether the conclusion is direct or indirect does not: the crucial question is whether the premises yield a conclusion (any conclusion) or not.

To the four traditional second-figure moods (Cesare, Camestres, Festino and Baroco), Buridan adds four more:

> "If the premises of Festino and Baroco are transposed, there will be two more moods concluding only indirectly, which can be called 'Tifesno' and 'Robaco', which are proved by reduction to Festino and Baroco simply by transposing the premises. The other two moods can only be concluded according to the non-normal construction, namely, if both premises are particular, one affirmative and the other negative." (Hubien, 1976, III i 4, p. 93)

To the six traditional third-figure moods (Darapti, Felapton, Disamis, Datisi, Bocardo and Ferison), he adds three more:

> "Darapti, Disamis and Datisi are valid for concluding both directly and indirectly; but Felapton, Bocardo and Ferison are only valid for concluding directly. But conversion of these three,

[17] See, e.g, Keynes (1884, III 4, §114).

[18] Aristotle identified four direct and two indirect moods in the first figure, four in the second figure and six in the third figure.

namely, Lapfeton, Carbodo and Rifeson, are valid for concluding only indirectly, and are reduced to direct ones by transposition of the premises." (Hubien, 1976, III i 4, p. 93)

We can explain the validity of the consequences here by using Buridan's account of supposition and of distribution. First, the conversion of the O-form 'Some *A* is not *B*': '*A*' has determinate supposition, and '*B*' has confused and distributive supposition, since we can descend both to

'This *A* is not *B* or that *A* is not *B* and so on for all *A*s'

(and ascend from any disjunct) and to

'Some *A* is not this *B* and some *A* is not that *B* and so on for all *B*s'

Buridan claims it converts accidentally to 'Every *B* (some) *A* is not': here too '*B*' has confused and distributive supposition, and '*A*' has determinate supposition, since we can descend as follows, both to

'This *B* *A* is not and that *B* *A* is not and so on for all *B*s'

and to

'Every *B* this *A* is not or every *B* that *A* is not and so on for all *A*s'

(and ascend from any disjunct). Thus whereas 'Every *B* is not *A*' is equivalent to

$$(\forall x)(Bx \to (\exists y)(Ay \land x \neq y)), \text{ i.e., } \bigwedge_i \bigvee_j B_i \neq A_j$$

'Every *B* (some) *A* is not' is equivalent to

$$(\exists y)(Ay \land (\forall x)(Bx \to x \neq y)), \text{ i.e., } \bigvee_j \bigwedge_i B_i \neq A_j$$

The "non-normal" conclusions in Buridan's extra syllogisms are all of the form '*SPo*', that is, 'Some *S* (some) *P* is not'. Here '*P*' has determinate supposition (whereas in '*SoP*', i.e., 'Some *S* is not *P*', '*P*' has confused and distributive supposition). Take this non-normal syllogism in the second figure:

PoM	Some *P* is not *M*
SiM	Some *S* is *M*
SPo	Some *S* (some) *P* is not

Only one premise is negative, and so is the conclusion (Buridan's condition 2). '*M*' is distributed at least once—namely, in the first premise (Buridan's condition 6). Neither term is distributed in the conclusion—they both have determinate supposition (Buridan's condition 8). So by Buridan's conclusions 2, 6 and 8, the syllogism is valid.

6 Oblique Terms

Another example of syllogisms containing propositions in non-standard form takes propositions with oblique terms (see Figure 3), e.g.:

> No human's donkey (i.e., no donkey of a human) is running
> A human is an animal
> So some animal's donkey is not running.[19]

An oblique term is any term in a proposition not in the nominative. The cases Buridan mostly considers are terms in the accusative and in the genitive, e.g.,

> 'Some human [it's] every horse [he or she] is seeing' (*Homo omnem equum est videns*—'*equum*' is in the accusative case)
>
> 'Every man's donkey is running' (*'Omnis asinus hominis currit*—'*hominis*' is in the genitive)

Buridan notes that sometimes the whole subject is distributed, as in the second example here; but sometimes only the term in the nominative, or in the genitive is, as in 'Of every man a donkey is running' (*Cuiuslibet hominis asinus currit*)—'man' has confused and distributive supposition but 'donkey' has determinate supposition The interesting syllogisms arise in this latter case, e.g., when the middle term is only part of the subject.

At each node of the octagon of propositions with oblique terms we find nine equivalent propositions (72 in all). E.g., at the top rightmost node we have:

- 'Of every human no donkey is running' (*Cuiuslibet hominis nullus asinus currit*)
- 'Of every human every donkey is not running' (*Cuiuslibet hominis quilibet asinus non currit*)
- 'Of every human not any donkey is running' (*Cuiuslibet hominis non quidam asinus currit*)
- 'Of no human [is it] not [that] no donkey is running' (*Nullius hominis non nullus asinus currit*)
- 'Of no human not every donkey is not running' (*Nullius hominis non quilibet asinus non currit*)
- 'Of no human is some donkey running' (*Nullius hominis aliquius asinus currit*)

[19] Cf. *Prior Analytics* I 36.

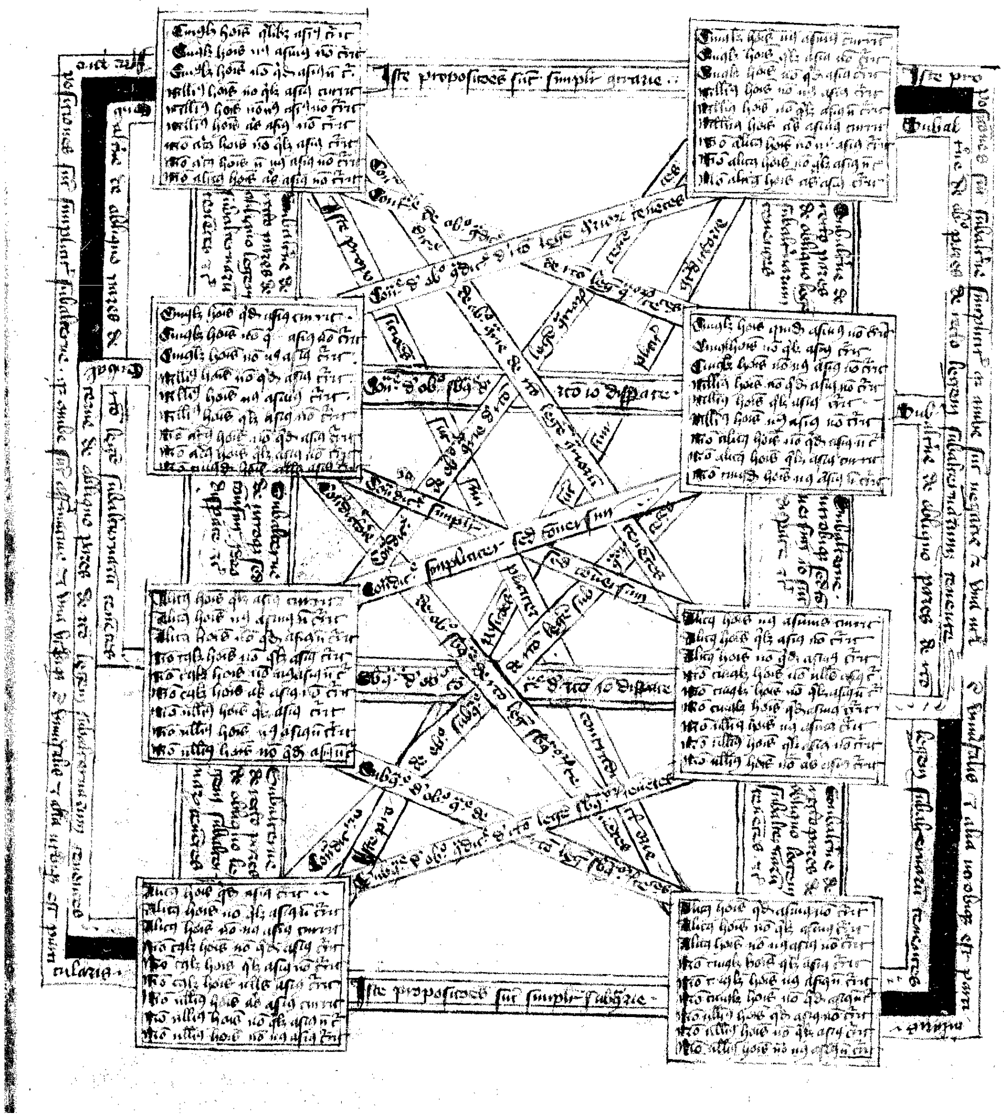
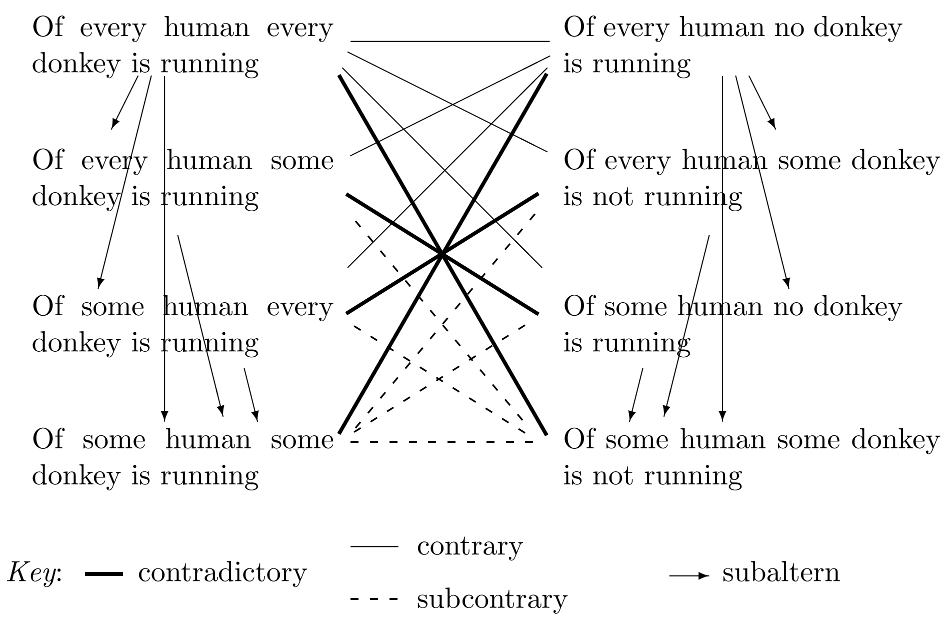


Figure 3: Buridan's Octagon of Opposition for Propositions containing Oblique Terms

- 'Not of any human is [it] not [that] no donkey is running' (*Non alicuius hominis non nullus asinus currit*)
- 'Not of any human is not every donkey not running' (*Non alicuius hominis non quilibet asinus non currit*)
- 'Not of any human is some donkey running' (*Non alicuius hominis aliquis asinus currit*)

There are 28 possible relations between the nodes ($= \frac{8!}{(8-2)!2!}$). Buridan's octagon with oblique terms draws the 24 logical relations which hold (there are also 4 disparate pairs):



Buridan proceeds to consider syllogisms with oblique terms:

> "Consider 'Every human's donkey is running' (i.e., 'Of every human some donkey is running'—*Cuiuslibet hominis asinus currit)*: 'human' is distributed and 'donkey' is not distributed, nor [is] the whole 'human's donkey' (*hominis asinus*). It would be the same if I said 'A donkey of every human is running'. For here 'donkey' supposits determinately, because no cause of confused [supposition] precedes it. Nonetheless, 'donkey' does not supposit alone but restricted by the distributed oblique [term] following it; so the said proposition would not be true if no donkeys were running or no donkey running were a donkey of any human." (Hubien, 1976, III ii 1, p. 99)

Just as the whole subject (e.g., 'human's donkey') may not be distributed, or have a single mode of supposition, so the subject may not be either extreme nor the middle term in a syllogism. In his *Treatise on Consequences*, Buridan's approach, like other medieval logicians, is to rephrase the proposition so that the middle term, or the extreme, appears as the subject in the nominative. E.g., 'Of some human a donkey is running' is equivalent to 'Some human is one of whom a donkey is running'.

But in the *Summulae*, Buridan seems happy to proceed without need for paraphrase. Consider:

> Of no human is a horse running (*Nullius hominis equus currit*)

Of every human an animal is running (*Cuiuslibet hominis animal currit*)
So of no human is (some) animal a human's horse (*Ergo nullius hominis animal est hominis equus*)

This is invalid, he claims, since 'animal' is distributed in the conclusion but was not distributed in the premises. As we saw in the octagon, 'Of no human is (some) animal ...' is equivalent to 'Of every human no animal ...', that is,

$$\bigwedge_i \neg \bigvee_j \phi(i,j) \equiv \bigwedge_i \bigwedge_j \neg\phi(i,j)$$

What does follow, Buridan observes, is

So of every human some animal is not a human's horse (*Ergo cuiuslibet hominis animal non est hominis equus*)

He writes:

"It is clear that in this syllogism, the middle term is 'running', the major extreme is 'human's horse', and the minor extreme is 'animal of a human', whatever you take to be the subject or the predicate in these propositions." (Buridan, 2001, V 8, pp. 368-9)

Recall De Morgan famous criticism of traditional logic as incapable of explaining the validity of the following inference:[20]

Every horse is an animal
So every horse's head is an animal's head.

Buridan's theory can show that this is valid. It's an enthymeme, not formally valid, but valid *simpliciter*, that is, absolutely valid. The suppressed premise is the necessary truth: 'Every horse's head is a horse's head'. Then the argument has this form:

Every M is a P	Every horse is an animal
Every horse's head is an M's head	Every horse's head is a horse's head
So every horse's head is a P's head	So every horse's head is an animal's head.

'M' is distributed in the first premise, but 'P' is not distributed in the conclusion, nor is 'M' in the second premise.

7 Modal Propositions

Buridan shows how the logical relations hold analogously between propositions with oblique terms and modal propositions (see Figure 4), e.g.,

[20]De Morgan (1966, pp. 29 and 216). Cf. Wengert (1974). Bochenski (1962, p. 95) claims that Aristotle himself could have shown the inference valid.

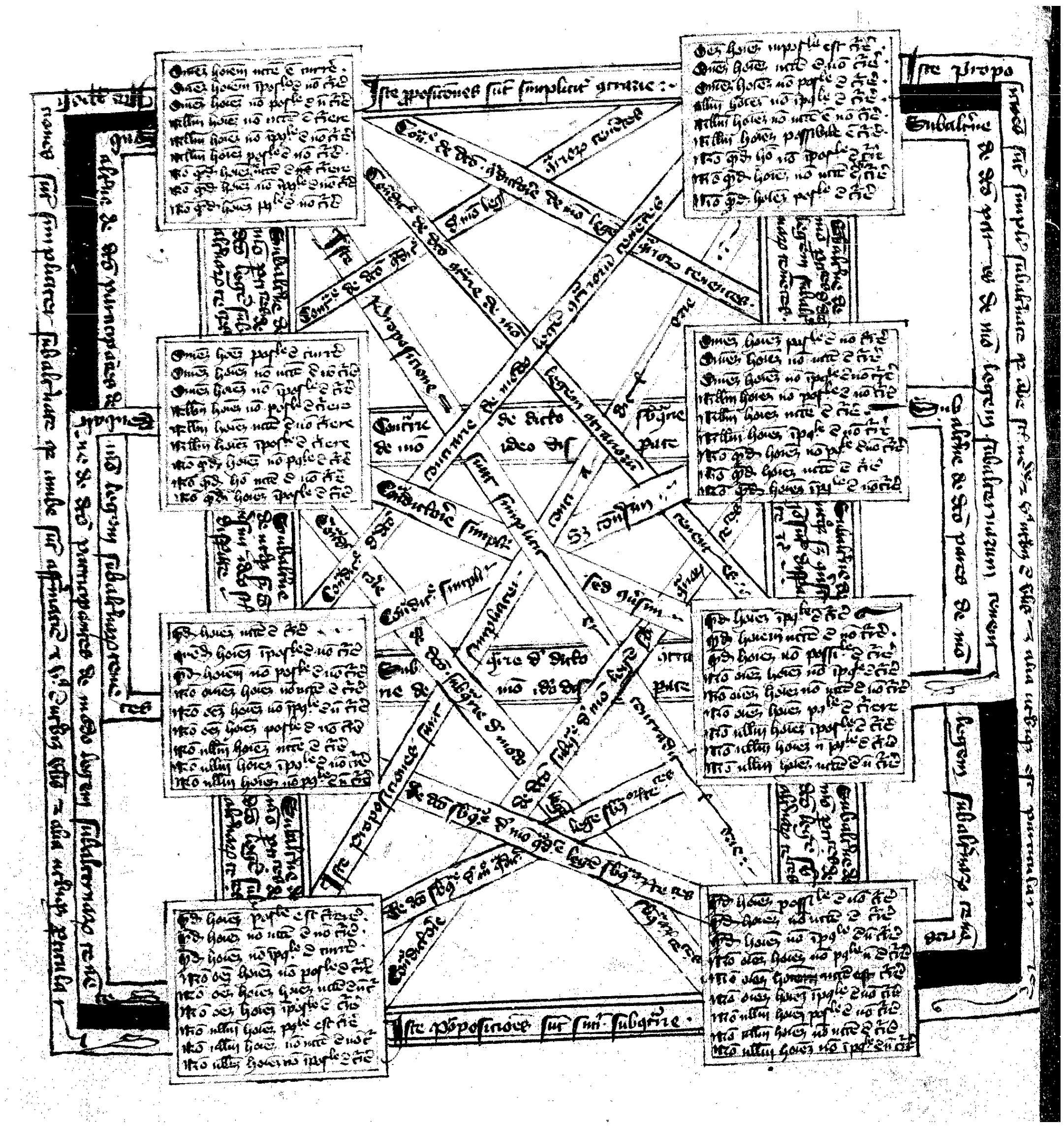


Figure 4: Buridan's Modal Octagon

Every human necessarily runs	is contrary to	Some human necessarily does not run

just as

Of every human every donkey runs	is contrary to	Of every human some donkey does not run

He writes:

> "We match the oblique term before the nominative in the propositions with oblique terms, and the subject term of the non-

normal propositions, with the subject term of modal propositions, and likewise, we match the nominative term of the propositions with oblique terms, and the predicate of the non-normal propositions, with the mode of modal propositions. In the three octagons, the careful observer will clearly see how the propositions with oblique terms distributable independently of their nominatives, or the non-normal propositions sharing both terms in the same order, are related to one another with respect to one or other of the laws of opposition." (Buridan, 2001, I 5, p. 43)

Klima presents Buridan's octagons in a composite table:[21]

	Modal	*Oblique*	*Non-normal*
1.	Every human necessarily runs	Of every human every donkey runs	Every human every runner is
2.	Every human possibly runs	Of every human some donkey runs	Every human some runner is
3.	Every human necessarily does not run	Of every human every donkey does not run	Every human every runner is not
4.	Every human possibly does not run	Of every human some donkey does not run	Every human some runner is not
5.	Some human necessarily runs	Of some human every donkey runs	Some human every runner is
6.	Some human possibly runs	Of some human some donkey runs	Some human some runner is
7.	Some human necessarily does not run	Of some human every donkey does not run	Some human every runner is not
8.	Some human possibly does not run	Of some human some donkey does not run	Some human some runner is not

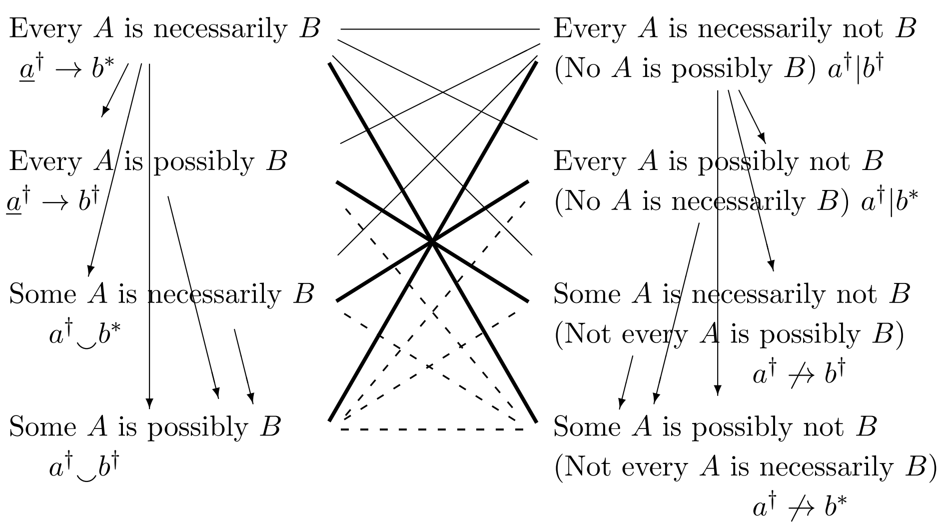
According to Buridan's analysis of modal propositions, all modal propositions are either taken in the composite sense (that is, *de dicto*), or in the divided sense (that is, *de re*). Nothing follows from composite modal premises where the mode is 'possible' or 'contingent'. From composite modal premises of necessity, the conclusion follows in the same mode. Buridan claims that all divided modal propositions ampliate their subject. Thus, e.g., 'Every A is necessarily B' is true iff everything which is or might be A must be B. Similarly, 'Some A is possibly B' is true iff something which is or might be A might be B. We can override this ampliation by replacing the subject by a 'what is'-phrase, e.g., 'Everything which is A is necessarily B'.

Paul Thom (2003, p. 17) uses the following key to represent the ampliation of terms and the forms of assertoric and modal categorical propositions:

[21] I have adapted this table from Klima's by interchanging nodes 2 and 5, and 4 and 7, in line with the mss.

'Every A is B':	$\underline{a} \to b$	$\underline{a}$:	indicates existential import
'No A is B':	$a \vert b$	$a^\dagger$:	what is A or possibly A
'Some A is B':	$a \smile b$ (a overlaps b, i.e., $\exists d$, $a \leftarrow d \to b$)	a^*:	what is necessarily A
'Not every A is B':	$a \not\to b$		

Using it, we can set out Buridan's modal octagon as follows:[22]



In a series of twenty-four theorems, Buridan takes us through all the various combinations of modal and assertoric propositions. E.g., Theorem 15 says:

> "In the first figure, from an assertoric major and a minor of necessity there does not follow a conclusion of necessity, or even assertoric except in Celarent" (Hubien, 1976, IV 2, p. 123)

In particular, Barbara XLL is invalid. Suppose God is, as it happens, creating:

> Every God is creating
> Any first cause is of necessity God
> But not every first cause is of necessity creating.

Celarent XLL is also invalid. But Celarent XLX is valid: $m|p$, $s^\dagger \to m^*$, so $s|p$. For $s \to s^\dagger \to m^* \to m|p$, so $s|p$.

However, Buridan also rejects Barbara LXL as invalid, by Theorem 16:

> "In the first figure, from a major of necessity and an assertoric minor there is always a valid syllogism to a particular conclusion of necessity, but not to a universal conclusion of necessity." (Hubien, 1976, IV 2, p. 124)

[22] Buridan's modal octagon was described at length in Hughes (1989).

The reason is Buridan's interpretation of necessity propositions as ampliating their subject. From $m^\dagger \to p^*$ and $s \to m$, $s^\dagger \to p^*$ does not follow. For something could be s which could not be m. For example, suppose only donkeys are white. All donkeys necessarily bray (if it could be a donkey, it is a donkey). But although on this assumption, all actual white things are donkeys, not everything which could be white (say, a horse) could be a donkey, so the conclusion ('All white things necessarily bray') is false.

8 Conclusion

Medieval logicians extended Aristotle's theory of the syllogism to a general theory of consequence. Within consequence, they distinguished formal from material consequence, and absolute from *ut nunc* consequence. They developed a theory of properties of terms, in particular, of signification, supposition and ampliation, which played a central role in their theory of consequence. Much of their work on the modal syllogism was an attempt to correct and clarify Aristotle's theory. John Buridan's theory of consequence and of the syllogism was the most sophisticated and extensive in its theoretical organisation. By the use of non-normal propositions, he was able to extend accidental conversin to allow O-propositions to convert, and to extend and systematize the assertoric syllogism. By giving a systematic account of propositions with oblique terms, he was able to show the validity of further consequences, such as De Morgan's notorious alleged counterexample. Finally, by making explicit Aristotle's brief remarks about the ampliation of terms in tensed and modal propositions, Buridan was able to give a more systematic account of the modal syllogism, and to exhibit a strong analogy between modal, oblique and non-normal propositions in his three octagons.

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